HW # 11

James Ding

Math N1A: Sections 4.5, 4.7, 4.9

Section 4.5

$c(x) = x^{1/3}(4+x)$

A. Domain: Made up of $\sqrt[3]{x}$ and $(4+x)$ which are both functions which have $\mathbb{R}$ as their domain.

$\boxed{\therefore c(x) \text{ domain is also } \mathbb{R}}$

B. x-intercepts

$0 = \sqrt[3]{x}\,(4+x)$

$\boxed{x = 0, -4}$

y-intercepts

$\boxed{y = 0}$

C. Symmetry

$f(-x) \neq -f(x) \text{ or } f(x)$

$\boxed{\text{Niether even or odd}}$

D. Asymptotes

Domain is $\mathbb{R}$, $\boxed{\therefore \text{no vert asymptotes}}$

$\lim_{x\to\infty} c(x) = \infty \qquad \lim_{x\to-\infty} c(x) = \infty$

$\boxed{\text{No horizontal asymptotes}}$

E. $c'(x) = \frac{d}{dx} x^{1/3}(4+x)$

$= x^{1/3} + (4+x)\left(\frac{1}{3}x^{-2/3}\right)$

$= \frac{3x}{3x^{2/3}} + \frac{4+x}{3x^{2/3}}$

$= \frac{4x+4}{3x^{2/3}}$

$= \frac{4(x+1)}{3x^{2/3}}$ Asymptote at $x=0$

Cut points: $-1, 0$

Interval	Sign	Direction
$(-\infty, -1)$	$-$	Decreasing
$(-1, 0)$	$+$	Increasing
$(0, \infty)$	$+$	Increasing

F. $c'(x) = \frac{4(x+1)}{3x^{2/3}}$ 0 isn't within domain

Critical Points: -1

$\boxed{\text{Local Low at } -1}$ Value is -3

G. $f''(x) = \frac{d}{dx}\frac{4x+4}{3x^{2/3}}$

$= \frac{(3x^{2/3})(4) - (4x-4)(2x^{-1/3})}{9x^{4/3}}$

$= \frac{12x^{2/3} - 8(x+1)x^{-1/3}}{9x^{4/3}}$

$\cancel{\therefore \frac{4(x+2)}{9x^{5/3}}}$

$= \frac{4(x-2)}{9x^{5/3}}$

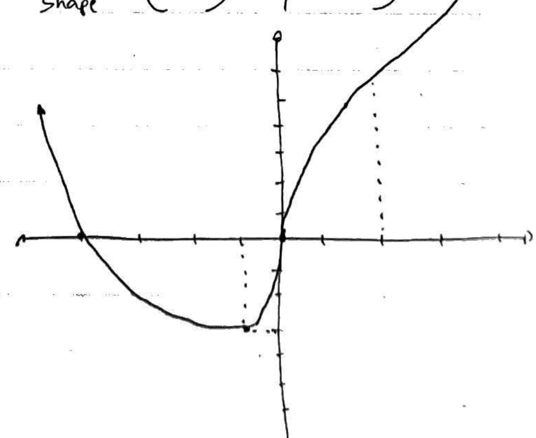
Cutpoints: 0, 2

Interval	Sign	Concavity
$(-\infty, 0)$	$+$	concave up
$(0, 2)$	$-$	concave down
$(2, \infty)$	$+$	concave up

H.

	-1		0		2	
$f'(x)$	$-$		$+$	$+$		$+$
$f''(x)$	$+$		$+$	$-$		$+$

Shape



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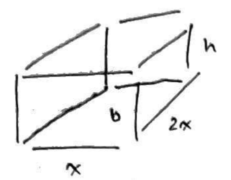
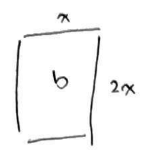




$A = 2x^2$

$V = 10m^3$

Let m_b be the multiplier for the base (10)

$V = 2x^2 h$

Let m_s be the mult. for sides (6)

$10 = 2x^2 h$

price must be positive

$h = \frac{10}{2x^2}$

$$p(x) = m_s\left(2(xh) + 2(2xh)\right) + m_b(2x^2)$$
$$= m_s(2xh + 4xh) + m_b(2x^2)$$
$$= 6m_s xh + 2x^2 m_b$$
$$= 6m_s x\left(\frac{10}{2x^2}\right) + 2x^2 m_b$$
$$= 2x^2 m_b + 6m_s\left(\frac{10}{2x}\right)$$
$$= 20x^2 + \frac{360}{2x}$$
$$= 20x^2 + \frac{180}{x}$$
$$= 20\left(x^2 + \frac{9}{x}\right)$$

$$p'(x) = \frac{d}{dx}\left[x^2 + \frac{9}{x}\right]$$
$$= 2x - \frac{9}{x^2}$$

undefined at 0

$$0 = 2x - \frac{9}{x^2} \;;\; x \in (0, \infty)$$
$$= \frac{2x^3}{x^2} - \frac{9}{x^2}$$
$$= 2x^3 - 9$$
$$x = \sqrt[3]{\frac{9}{2}} \approx 1.65$$

~~Interval~~	~~Sign~~
~~$(-\infty, \sqrt[3]{\frac{9}{2}})$~~	
~~$(\sqrt[3]{\frac{9}{2}}, \infty)$~~	

Interval	Sign
$(0, \sqrt[3]{\frac{9}{2}})$	$-$
$(\sqrt[3]{\frac{9}{2}}, \infty)$	$+$

} Minimum exists

Back to top

Minimum exists at $x = \sqrt[3]{\frac{9}{2}}$

$$P\left(\sqrt[3]{\frac{9}{2}}\right) = 20\left(\sqrt[3]{\frac{9}{2}}\right)^2 + \frac{180}{\sqrt[3]{\frac{9}{2}}}$$

$$\boxed{\approx \$163.541}$$

21. $y = 2x + 3$

$$d(x) = \sqrt{(x-0)^2 + (y-0)^2}$$
$$= \sqrt{x^2 + y^2}$$
$$= \sqrt{x^2 + (2x+3)^2}$$
$$= \sqrt{x^2 + 4x^2 + 12x + 9}$$
$$= \sqrt{5x^2 + 12x + 9}$$

Will have same minimum as $5x^2 + 12x + 9$

Find minimum of $d(x)^2$

$$\frac{d}{dx}\left[5x^2 + 12x + 9\right]$$
$$= 10x + 12$$

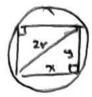
Follows $mx + b$, only has one zero.

$x = -1.2$

$y = 2(-1.2) + 3 = -2.4 + 3 = 0.6$

$$\boxed{(-1.2, 0.6)}$$

25.



$x^2+y^2=(2r)^2$ $r>0$

$y=\sqrt{(2r)^2-x^2}$

$A=xy$

$=x\sqrt{(2r)^2-x^2}$; $x\in(0,\infty)$ Area / Length cannot be a negtive or zero

$=\sqrt{4r^2x^2-x^4}$

Find max of A^2 in $x\in(0,\infty)$

$\frac{d}{dx}\left[4r^2x^2-x^4\right]$

$=8r^2x-4x^3$

$=-4x(x^2-2r^2)$

$0=-4x(x^2-2r^2)$ $x\in[0,\infty)$

$0=x^2-2r^2$ $0=-4x$

$x^2=2r^2$ $x=0$

$x=r\sqrt{2}$

Interval	Sign	
$(0, r\sqrt{2})$	+	} Max occurs
$(r\sqrt{2}, \infty)$	−	

Max occurs at $x=\sqrt{2}\cdot r$

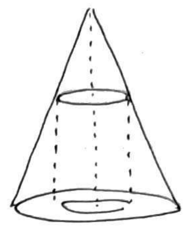
$(r\sqrt{2})^2+y^2=(2r)^2$

$2r^2+y^2=4r^2$

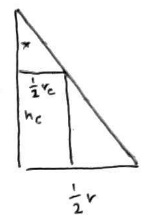
$y=2r^2$ Equal to x, meaning the rectangle is a square

$\boxed{\text{Max dimensions are each side being } \sqrt{2}\cdot r}$

32.



Cross Section



$V=\pi r_c^2 h_c$

$r_c\in(0,\infty)$

$\frac{h}{\frac{1}{2}r}=\frac{x}{\frac{1}{2}r_c}$

$\frac{h}{r}=\frac{x}{r_c}$

$rx=hr_c$

$r(h-h_c)=hr_c$

$rh-rh_c=hr_c$

$rh_c=rh-hr_c$

$h_c=\frac{rh-hr_c}{r}$

$V=\pi r_c^2\left(\frac{rh-hr_c}{r}\right)$

$=\frac{\pi r_c^2 rh}{r}-\frac{\pi r_c^3 h r_c}{r}$

$=\pi r_c^2 h-\frac{\pi r_c^3 h}{r}$

$=\pi h r_c^2-\frac{\pi h}{r}r_c^3$

$=\frac{d}{dr_c}\left[-\frac{\pi h}{r}r_c^3+\pi h r_c^2\right]$

$=-\frac{3\pi h}{r}r_c^2+2\pi h r_c$

$=-\pi h r_c\left(\frac{3}{r}r_c-2\right)$ r_c cannot be 0

$0=\frac{3}{r}r_c-2$

$r_c=\frac{2r}{3}$

Interval	Sign
$(0, \frac{2r}{3})$	+
$(\frac{2r}{3}, \infty)$	−

$$\boxed{V=\pi\left(\frac{2r}{3}\right)\left(\frac{rh-h\left(\frac{2r}{3}\right)}{r}\right)}$$

Section 4.9

12. $\sqrt[3]{x^2} + x\sqrt{x}$

$$F(x) = \frac{x^{5/3}}{\frac{5}{3}} + \frac{x^{5/2}}{\frac{5}{2}}$$

$$\boxed{= \frac{3}{5}x^{5/3} + \frac{2}{5}x^{5/2} + c}$$

$$F'(x) = x^{\frac{3}{2}} + x^{\frac{2}{3}}$$ ✓

~~16. [illegible]~~

16. $r(\theta) = \sec\theta \tan\theta - 2e^{\theta}$

$$\boxed{R(\theta) = \sec\theta - 2e^{\theta} + c}$$

$$R'(\theta) = \sec\theta \tan\theta - 2e^{\theta}$$

25. $f''(x) = 20x^3 - 12x^2 + 6x$

$$f'(x) = \frac{20x^4}{4} - \frac{12x^3}{3} + \frac{6x^2}{2} + c$$

$$= 5x^4 - 4x^3 + 3x^2 + c$$

$$f(x) = \frac{5x^5}{5} - \frac{4x^4}{4} + \frac{3x^3}{3} + cx + D$$

$$\boxed{= x^5 - x^4 + x^3 + cx + D}$$

36. $f'(x) = \frac{(x+1)}{\sqrt{x}}$

$$f(x) = \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + c$$

$$= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$$

$$5 = \frac{2}{3}(1) + 2(1) + c$$

$$c = \frac{7}{3}$$

$$\boxed{f(x) = \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + \frac{7}{3}}$$

39. $f''(x) = -2 + 12x - 12x^2$

$$f'(x) = -2x + \frac{12x^2}{2} - \frac{12x^3}{3} + c$$

$$12 = -2(0) + \frac{12(0)}{2} - \frac{12(0)}{3} + c$$

$$c = 12$$

$$f'(x) = -2x + 6x^2 - 4x^3 + 12$$

$$f(x) = -\frac{2x^2}{2} + \frac{6x^3}{3} - \frac{4x^4}{4} + D + 12x$$

$$4 = -\frac{2(0)}{2} + \frac{6(0)}{3} - \frac{4(0)}{4} + D + 12(0)$$

$$D = 4$$

$$\boxed{f(x) = -x^2 + 2x^3 - x^4 + 12x + 4}$$

52. a. Slope is positive; like very steep in the beginning, meaning that it cannot be b.

Towards the middle, f crosses 0. This means a local max occurs there. ~~[illegible]~~ Only a has that attribute. ∴ It must be a.