HW#12

Math N1A: Sections 4.9, 5.1, 5.2, 5.3

Section 4.9

62. $a(t) = 3\cos t - 2\sin t$

$v(t) = 3\sin t + 2\cos t + C$

$s(t) = -3\cos t + 2\sin t + Ct + D$

$4 = 3\sin 0 + 2\cos 0 + C$ $\quad\quad$ $0 = -3\cos 0 + 2\sin 0 + C\cdot 0 + D$

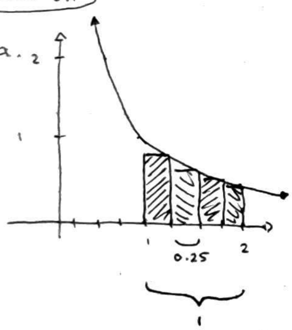
$= 2 + C$ $\quad\quad$ $= -3 + D$

$C = 2$ $\quad\quad$ $D = 3$

$$\boxed{s(t) = -3\cos t + 2\sin t + 2t + 3}$$

Section 5.1

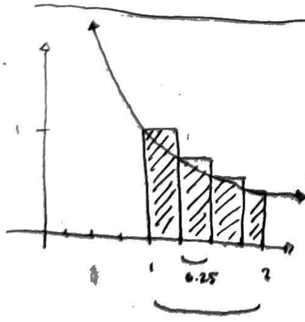
3. a.



$$A = \frac{1}{4}\left(\frac{4}{5} + \frac{2}{3} + \frac{4}{7} + \frac{1}{2}\right)$$

$$\boxed{= \frac{533}{840} \text{ units}^2 \approx 0.6345 \text{ units}^2 \quad \text{Underestimate}}$$

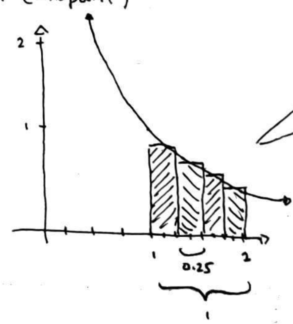
b.

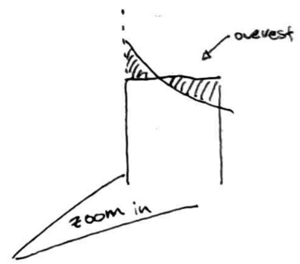


$$A = \frac{1}{4}\left(1 + \frac{4}{5} + \frac{2}{3} + \frac{4}{7}\right)$$

$$\boxed{= \frac{319}{420} \text{ units}^2 \approx 0.7595 \text{ units}^2 \quad \text{Over estimate}}$$

c. (Midpoint)

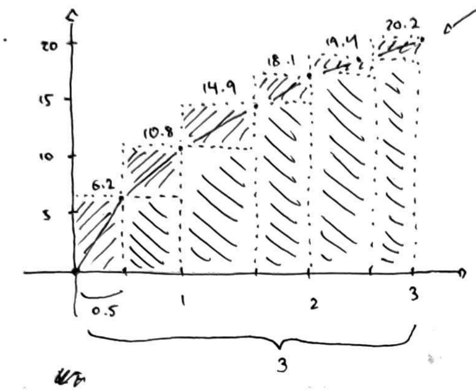




$$A = \frac{1}{4}\left(\frac{8}{9} + \frac{8}{11} + \cancel{\frac{13}{8}} + \frac{8}{13} + \frac{8}{15}\right)$$

$$\boxed{= \frac{4448}{6435} \text{ units}^2 \approx 0.69122 \text{ units}^2 \quad \text{Over estimate}}$$

13.



note: not actually a precise line (just to give a rough shape of v(t))

$$L = \frac{1}{2}\left(6.2 + 10.8 + 14.9 + 18.1 + 19.4\right)$$

$$= 34.7 \text{ ft}$$

$$U = \frac{1}{2}\left(6.2 + 10.8 + 14.9 + 18.1 + 19.4 + 20.2\right)$$

$$= 44.8 \text{ ft}$$

Lower est: 34.7 ft
Upper est: 44.8 ft

21. $$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{b-a}{n}\right) f\left(a + \frac{i(b-a)}{n}\right)$$

Substitute:

$$\boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n}\right)\left(\frac{2\left(1+\frac{2i}{n}\right)}{\left(1+\frac{2i}{n}\right)^2+1}\right)}$$

Section 5.2

18. $$\lim_{n\to\infty} \sum_{i=1}^{n} x_i\sqrt{1+x_i^3}\,\Delta x \quad [2,5]$$

$$\boxed{= \int_2^5 x\sqrt{1+x^3}\,dx}$$

21. $$\int_2^5 (4-2x)\,dx$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} (4-2x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(4-2\left(a+i\left(\frac{b-a}{n}\right)\right)\right)\left(\frac{b-a}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(4-2\left(2+i\left(\frac{3}{n}\right)\right)\right)\left(\frac{3}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(-2i\left(\frac{3}{n}\right)\left(\frac{3}{n}\right)\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{-18i}{n^2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{-18}{n^2}\right)\left(\sum_{i=1}^{n} i\right)$$

$$= \lim_{n\to\infty} \left(\frac{-18}{n^2}\right)\left(\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{-9(n+1)}{n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{-9n-9}{n}\right) = \boxed{-9}$$

25. $$\int_0^1 (x^3-3x^2)\,dx$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} (x_i^3 - 3x_i^2)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(a+\frac{i(b-a)}{n}\right)^3 - 3\left(a+\frac{i(b-a)}{n}\right)^2\right)\left(\frac{b-a}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{i^3}{n^3} - 3\frac{i^2}{n^2}\right)\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \frac{1}{n}\left(\sum_{i=1}^{n} \frac{i^3}{n^3} - \sum_{i=1}^{n} 3\frac{i^2}{n^2}\right)$$

$$= \lim_{n\to\infty} \frac{1}{n}\left(\frac{1}{n^3}\sum_{i=1}^{n} i^3 - 3\frac{1}{n^2}\sum_{i=1}^{n} i^2\right)$$

$$= \lim_{n\to\infty} \frac{1}{n}\left(\frac{1}{n^3}\cdot\frac{n^2(n+1)^2}{4} - \frac{3}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty} \frac{1}{n}\left(\frac{(n+1)^2}{4n} - \frac{3(n+1)(2n+1)}{6n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{(n+1)^2}{4n^2} - \frac{(n+1)(2n+1)}{2n^2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{-(n+1)(3n+1)}{4n^2}\right) = \lim_{n\to\infty} \left(\frac{-3n^2-4n-1}{4n^2}\right) = \boxed{-\frac{3}{4}}$$

34. a. $\int_0^2 g(x)\,dx = \frac{1}{2}(2-0)(4-0)$

$\boxed{=4}$

b. $\int_2^6 g(x)\,dx = -\frac{1}{2}\pi 2^2$

$\boxed{=-2\pi}$

c. $\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx$

$= 4 + (-2\pi) + \frac{1}{2}(1)(1)$

$\boxed{= \frac{9}{2} - 2\pi}$

~~36. area of circle with~~

37. $\frac{1}{4}$ area of circle with $r=3$ + 3 units

$= \frac{1}{4}\pi \cdot 9 + 3$

$\boxed{= \frac{9}{4}\pi + 3}$

54. ??? $m \le f(x) \le M$; $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$

57. Min of $\sqrt{1+x^2}$ $[-1,1]$: 1

Max of $\sqrt{1+x^2}$ $[-1,1]$: $\sqrt{2}$

$1(1-(-1)) \le \int_{-1}^{1} \sqrt{1+x^2}\,dx \le \sqrt{2}\,(1-(-1))$

$\hookrightarrow \boxed{2 \le \int_{-1}^{1} \sqrt{1+x^2}\,dx \le 2\sqrt{2}}$

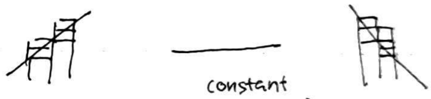
73. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5}$

~~$\lim_{n\to\infty} \frac{1}{n^5} \sum_{i=1}^{n} i^4$~~

$\boxed{\int_0^1 x^4\,dx}$

Luvreet's Problems

1. True. let $[a,b]$ be a situation where $f(x)$ has the following behaviors



$\boxed{\text{No situation has all underest.}}$

2. True. No height means area = 0.

3. False. odd funcs can cancel out

$\int_{-1}^{1} x^3\,dx$  $= 0$

4. True. This allows for positive slope only, which creates area, or 0, which remains constant.

5. ??