Section 5.3

28. $\int_0^4 (4-t)\sqrt{t}\, dt$

$f(t) = (4-t)\sqrt{t}$

$= -t\sqrt{t} + 4\sqrt{t}$

$= -t^{3/2} + 4t^{1/2}$

$F(t) = -\frac{2}{5}t^{5/2} + \frac{8}{3}t^{3/2}$

$\int_0^4 f(t)dt = F(t)\Big|_0^4$

$= -\frac{2}{5}(4)^{3/2} + 4(4)^{1/2}$

$= -\frac{2}{5}(32) + \frac{8}{3}(8)$

$= -\frac{64}{5} + \frac{64}{3}$

$= \frac{320-192}{15} = \boxed{\frac{128}{15} \approx 8.533}$

35. $\int_1^2 \frac{v^3 + 3v^6}{v^4} dv$

$f(v) = \frac{v^3+3v^6}{v^4}$

$= v^{-1} + 3v^6$

$F(v) = \ln|v| + \frac{3v^3}{3}$

$= \ln|v| + v^3$

$\int_1^2 f(v)\,dv = F(v)\Big|_1^2$

$= \ln(2) + 8 - (\ln(1) + 1)$

$\boxed{= \ln(2) + 7 \approx 7.693}$

~~61. F(x) = ∫ from √x to 2x~~

37. $\int_0^1 (x^e + e^x)\, dx$

$f(x) = x^e + e^x$

$F(x) = \frac{x^{e+1}}{e+1} + e^x$

$\int_0^1 f(x)dx = F(x)\Big|_0^1$

$= \left(\frac{1^{e+1}}{e+1} + e\right) - (0+1)$

$\boxed{= \frac{1}{e+1} + e - 1}$

39. $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{8}{1+x^2} dx$

$= 8\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{1}{1+x^2}dx$; $f(x) = \frac{1}{1+x^2}$; $F(x) = \tan^{-1}x$

$= 8\left(F(x)\Big|_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}\right)$

$= 8\left(\tan^{-1}\sqrt{3} - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$

$= 8\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$

$= \frac{8\pi}{6}$

$\boxed{= \frac{4\pi}{3}}$

62. $F(x) = \int_{\sqrt{x}}^{2x} \tan^{-1}t\, dt$

let $u = 2x$

$F(x) = \int_{\sqrt{x}}^{u} \tan^{-1}t\, dt$

$f(x) = \frac{d}{dx}\tan^{-1}u$

$= \frac{d}{dx}\tan^{-1}2x$

$= \frac{1}{1+4x^2}\cdot 2$

$\boxed{= \frac{2}{1+4x^2}}$

43. $\int_0^{\pi} f(x)dx$

$= \int_0^{\frac{\pi}{2}} \sin x\,dx + \int_{\frac{\pi}{2}}^{2\pi} \cos x\, dx$

$g(x) = \sin x$; $G(x) = -\cos x$ $\quad h(x) = \cos x$; $H(x) = \sin x$

$= G(x)\Big|_0^{\frac{\pi}{2}} + H(x)\Big|_{\frac{\pi}{2}}^{2\pi}$

$= (-\cos\frac{\pi}{2} + \cos 0) + (\sin 2\pi - \sin\frac{\pi}{2})$

$= 1 - 1$

$\boxed{= 0}$

57. $f(x) = \sec x \tan x$

$= \frac{\sin x}{\cos^2 x}$

$\cos^2 x \neq 0$; $x \neq \frac{\pi}{2} + \pi k$

Through the interval, $[\frac{\pi}{3}, \pi]$ $f(x)$ is undefined at $\frac{\pi}{2}$. This shows that $f(x)$ isn't continuous and you cannot use FTC 2

73. a. Local Max: 1, 5, 9

Local Min: 3, 7

b. when $x = 9$

c. $(\frac{1}{2}, 2) \cup (4,6) \cup (8,9)$

Section 5.4

14. $\int \left(\frac{1+v}{v}\right)^2 dv$

$f(v) = \left(\frac{1+v}{v}\right)^2$

$= \frac{(v+1)^2}{v^2}$

$= \frac{v^2+2v+1}{v^2}$

$= \frac{v^2}{v^2} + \frac{2v}{v^2} + \frac{1}{v^2}$

$\boxed{F(v) = v + 2\ln|v| - \frac{1}{v}}$

37. $\int_0^{\frac{\pi}{4}} \frac{1+\cos^2\theta}{\cos^2\theta} d\theta$

$f(\theta) = \frac{1+\cos^2\theta}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta} + 1$

$= \sec^2\theta + 1$

$F(\theta) = \tan\theta + \theta$

$\int_0^{\frac{\pi}{4}} f(\theta)d\theta = F(\theta)\Big|_0^{\frac{\pi}{4}}$

$= \tan\frac{\pi}{4} + \frac{\pi}{4} - (\tan 0 + 0)$

$\boxed{= \tan\frac{\pi}{4} + \frac{\pi}{4}}$

60. $v(t) = t^2 - 2t - 3$

a. $\int_2^4 v(t)\,dt$

$s(t) = \frac{t^3}{3} - t^2 - 3t$

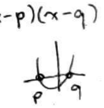
$\int_2^4 v(t)dt = s(t)\Big|_2^4$

$= \left(\frac{4^3}{3} - 4^2 - 3(4)\right) - \left(\frac{2^3}{3} - 2^2 - 3(2)\right)$

$\boxed{= \frac{2}{3}}$

b. $v(t) = t^2 - 2t - 3$

$= (t-3)(t+1) \leftarrow$ follows form of $(x-p)(x-q)$ which looks like



$v(t) \le 0$ when $t \in [-1, 3]$

Displacement: $\int_2^3 -v(t)dt + \int_3^4 v(t)dt$

$= -s(t)\Big|_2^3 + s(t)\Big|_3^4$

$= -\left[\left(\frac{3^3}{2} - 3^2 - 3(3)\right) - \left(\frac{2^3}{2} - \cancel{8}\, 2^2 - 3(2)\right)\right] + \left[\left(\frac{4^3}{2} - 4^2 - 3(4)\right) - \left(\frac{3^3}{2} - 3^3 - 3(3)\right)\right]$

$= \frac{5}{3} + \frac{7}{3}$

$\boxed{= 4}$

67. $C'(x) = 3 - 0.01x + 0.000006x^2$

$C(x) = 3x - 0.01x^2/2 + 0.000006x^3/3$

$C(x)\Big|_{2000}^{4000} = \left[3(4000) - \frac{0.01(4000)^2}{2} + \frac{0.000006(4000)^3}{3}\right] - \left[3(2000) - \frac{0.01(2000)^2}{2} + \frac{0.000006(2000)^3}{3}\right]$

$\boxed{= 58{,}000}$

Section 5.5

7. $\int x\sqrt{1-x^2}\,dx$

let $u = 1-x^2$

$\frac{du}{dx} = -2x$

$dx = \frac{du}{-2x}$

$= x\sqrt{u}\left(\frac{du}{-2x}\right)$

$= \frac{\sqrt{u}}{-2}\,du$

$= \frac{u^{1/2}}{-2}\,du$

$= \frac{u^{3/2}}{-2\left(\frac{3}{2}\right)} + C$

$\boxed{= \frac{(1-x^2)^{3/2}}{-3} + C}$

31. $\int \frac{(\arctan x)^2}{x^2+1}\,dx$

let $u = \arctan x$

$\frac{du}{dx} = \frac{1}{x^2+1}$

$dx = du\,(x^2+1)$

$= \frac{u^2\,(x^2+1)}{x^2+1}\cdot du$

$= u^2\,du$

$= \frac{u^3}{3} + C$

$\boxed{= \frac{(\arctan x)^3}{3} + C}$

33. $\int \cos(1+5t)\,dt$

let $u = 1+5t$

$\frac{du}{dt} = 5$

$dt = \frac{du}{5}$

$= \cos(u)\cdot\frac{du}{5}$

$= \frac{\cos(u)}{5}\,du$

$= \frac{\sin(u)}{5} + C$

$\boxed{= \frac{\sin(1+5t)}{5} + C}$

41. $\int \cot x\,dx$

$= \int \frac{1}{\tan x}\,dx$

$= \int \frac{\cos x}{\sin x}\,dx$

let $u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$= \frac{\cos x}{u\cos x}\,du$

$= \frac{1}{u}\,du$

$= \ln|u| + C$

$\boxed{= \ln|\sin x| + C}$

45. $\int \frac{1+x}{1+x^2}\,dx$

$= \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx$

$= \tan^{-1}x + \int \frac{x}{1+x^2}\,dx$

let $u = 1+x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\frac{x}{u\,2x}\,du$

$= \frac{1}{u\cdot 2}\,du$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2+1| + C$

$\boxed{\frac{1}{2}\ln|x^2+1| + \tan^{-1}x + C}$

48. $\int x^3\sqrt{x^2+1}\,dx$

let $u = x^2+1$ $\qquad x^2 = u-1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$= x^3\sqrt{u}\cdot\frac{du}{2x}$

$= \frac{x^2\sqrt{u}}{2}\,du$

$= \frac{(u-1)\sqrt{u}}{2}\,du$

$= \frac{u^{3/2} - u^{1/2}}{2}\,du$

$= \frac{u^{3/2}}{2}\cdot du - \frac{u^{1/2}}{2}\,du$

$= \frac{u^{5/2}}{2\left(\frac{5}{2}\right)} - \frac{u^{3/2}}{2\left(\frac{3}{2}\right)} + C$

$= \frac{u^{5/2}}{5} - \frac{u^{3/2}}{3} + C = \boxed{\frac{(x^2+1)^{5/2}}{5} - \frac{(x^2+1)^{3/2}}{3}}$

59. $\int \frac{e^{1/x}}{x^2}\,dx$

let $u = \frac{1}{x}$

$\frac{du}{dx} = -\frac{1}{x^2}$

$dx = -x^2\,du$

$= \frac{e^u(-x^2)}{x^2}\,du$

$= -e^u\,du$

$= -e^u + c \quad = -e^{\frac{1}{x}} + c$

$-e^{\frac{1}{x}}\Big|_1^2 = e^{\frac{1}{x}}\Big|_2^1 = \boxed{e - e^{\frac{1}{2}}}$

73. ??

86 ??

Luvveet's Problems

69. $\int \frac{1}{x\sqrt{\ln x}}\,dx$

let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$\frac{1}{x\sqrt{u}} \cdot x\,du$

$= \frac{1}{\sqrt{u}}\,du$

$= u^{-\frac{1}{2}}\,du$

$= \frac{\sqrt{u}}{\frac{1}{2}}$

$= 2\sqrt{u}$

$2\sqrt{\ln x}\Big|_e^{e^4} = 2\sqrt{4} - 2\sqrt{1} = 4 - 2 = \boxed{2}$

1. $\int \frac{2x}{1+x^2}\,dx$

let $u = 1+x^2$

$\frac{du}{dx} = 2x$

$= \frac{1}{u}\,du$

$= \ln|u| + C$

$\boxed{= \ln|x^2+1| + C}$

2. Not sure

3. Not sure.