HW #2

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Math N1A: Sections 2.1–2.3

Section : 2.1

3. a. let $f(x) = \frac{1}{1-x}$

i. $\frac{f(1.5)-f(2)}{1.5-2} = \boxed{2}$

ii. $\frac{f(1.9)-f(2)}{1.9-2} = \boxed{1.111111\ldots}$

iii. $\frac{f(1.99)-f(2)}{1.99-2} = \boxed{1.010101\ldots}$

iv. $\frac{f(1.999)-f(2)}{1.999-2} = \boxed{1.001001\ldots}$

v. $\frac{f(2.5)-f(2)}{2.5-2} = \frac{2}{3} = \boxed{0.666667}$

vi. $\frac{f(2.1)-f(2)}{2.1-2} = \boxed{0.909091\ldots}$

vii. $\frac{f(2.01)-f(2)}{2.01-2} = \boxed{0.990099\ldots}$

viii. $\frac{f(2.001)-f(2)}{2.001-2} = \boxed{0.999001\ldots}$

b. $\boxed{1}$

c. $y = mx + b$

$-1 = (1)(2) + b$

$\Rightarrow -1 = 2 + b$

$b = -3$

$\boxed{\therefore\ y = x - 3}$

5. a. let $f(x) = -16t^2 + 40t$

$f(2) = -16(2)^2 + 40(2)$

$= -64 + 80$

$= 16$

i. $\frac{f(2.5)-f(2)}{0.5}$

$= \frac{-16(2.5)^2 + 40(2.5) - 16}{0.5}$

$\boxed{= -32}$

ii. $\frac{f(2.1)-f(2)}{0.1}$

$= \frac{-16(2.1)^2 + 40(2.1) - 16}{0.1}$

$\boxed{= -25.6}$

iii. $\frac{f(2.05)-f(2)}{0.05}$

$= \frac{-16(2.05)^2 + 40(2.05) - 16}{0.05}$

$\boxed{= -24.8}$

iv. $\frac{f(2.01)-f(2)}{0.01}$

$= \frac{-16(2.01)^2 + 40(2.01) - 16}{0.05}$

$\boxed{= -24.16}$

5. b. $\boxed{-24 \text{ ft/s}}$

Section: 2.2

2. When $x$ approaches 1 from the left on a graph, it gets closer to 3. On the right of 1, the graph gets closer to 7.

$\lim_{x\to1} f(x)$ will not exist, as both $\lim_{x\to1^-} f(x)$ and $\lim_{x\to1^+} f(x)$ need to be the same.

6. a. $\boxed{4}$

b. $\boxed{4}$

c. $\boxed{4}$

d. $\boxed{\text{DNE}}$ Point is not filled in, and a solid point does not exist.

e. $\boxed{1}$

f. $\boxed{-1}$

g. $\boxed{\text{DNE}}$. $\lim_{x\to0^-} h(x)$ must be the same as $\lim_{x\to0^+} h(x)$

h. $\boxed{1}$

i. $\boxed{2}$

j. $\boxed{\text{DNE}}$ No point at $x=2$ is filled in, meaning an answer does not exist.

k. $\boxed{3}$

l. $\boxed{\text{DNE}}$ As $x$ gets closer to 5 (from the left), the graph fluctuates to the point where the answer lies in a range of numbers, in this case, 2 and 4.

8. a. $\boxed{\text{DNE} : \infty}$

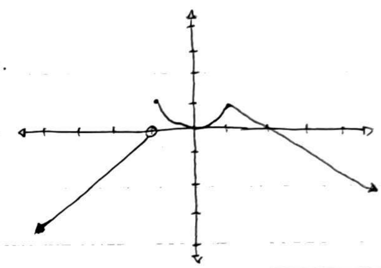
b. $\boxed{\text{DNE} : -\infty}$

c. $\boxed{\text{DNE} : \infty}$

d. $\boxed{\text{DNE} : -\infty}$

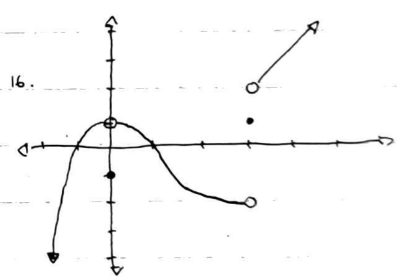
e. $\boxed{x = -3,\ x = -1,\ x = 2}$

11.



$\boxed{\lim_{x\to a} f(x) \text{ exists when } a \text{ is not equal to } -1}$

16.



2.2

a. $\lim_{x\to4} \frac{\sqrt{x}}{4-x}$

$\lim_{x\to4^-} \frac{\sqrt{x}}{4-x} = \frac{2}{0^+} = \infty$

$\lim_{x\to4^+} \frac{\sqrt{x}}{4-x} = \frac{2}{0^-} = -\infty$

$\lim_{x\to4^-} \frac{\sqrt{x}}{4-x} \neq \lim_{x\to4^+} \frac{\sqrt{x}}{4-x}$

$\boxed{\therefore\ \text{DNE}}$

b. $\lim_{x\to4} \frac{\sqrt{x}}{16-8x+x^2}$

$\lim_{x\to4^-} \frac{\sqrt{x}}{16-8x+x^2} = \frac{2}{0^+} = \infty$

$\lim_{x\to4^+} \frac{\sqrt{x}}{16-8x+x^2} = \frac{2}{0^+} = \infty$

$\boxed{\therefore\ \text{DNE} : \infty}$

38. $\lim_{x\to\pi^-} \cot(x)$

$= \lim_{x\to\pi^-} \frac{1}{\tan x}$

$= \lim_{x\to\pi^-} \frac{1}{0^-} = -\infty$

$\boxed{= \text{DNE}: -\infty}$ ■

**Section 2.3**

2. a. $\lim_{x\to 2} [f(x) + g(x)]$

$= \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x)$

$= (-1) + 2$

$\boxed{= 1}$ ■

b. $\lim_{x\to 0} [f(x) - g(x)]$

$= \lim_{x\to 0} f(x) - \lim_{x\to 0} g(x)$

$= 2 - \text{DNE}$

$\boxed{= \text{DNE}}$ ■ (Both limits must exist)

c. $\lim_{x\to -1} [f(x)\,g(x)]$

$= \lim_{x\to -1} f(x) \cdot \lim_{x\to -1} g(x)$

$= (1)\cdot(2)$

$\boxed{= 2}$ ■

d. $\lim_{x\to 3} \frac{f(x)}{g(x)}$

$= \frac{\lim_{x\to 3} f(x)}{\lim_{x\to 3} g(x)}$

$= \frac{1}{0} \rightarrow \lim_{x\to 3^-} \frac{f(x)}{g(x)} = \frac{1}{0^+}, \lim_{x\to 3^+} \frac{f(x)}{g(x)} = \frac{1}{0^-}$

$\boxed{= \text{DNE}}$ ■ (Denominator cannot be 0)

(Left and rt. limits are not equal)

2. e. $\lim_{x\to 2} [x^2 f(x)]$

$= \lim_{x\to 2} x^2 \cdot \lim_{x\to 2} f(x)$

$= 4 \cdot (-1)$

$\boxed{= -4}$ ■

f. $f(-1) + \lim_{x\to -1} g(x)$

$= 3 + 2$

$\boxed{= 5}$ ■

12. $\lim_{x\to -3} \frac{x^2 + 3x}{x^2 - x - 12}$

$= \lim_{x\to -3} \frac{x(x+3)}{(x-4)(x+3)}$

$= \lim_{x\to -3} \frac{x}{x-4}$

$= \lim_{x\to -3} \frac{-3}{-3-4}$

$\boxed{= \frac{3}{7}}$ ■

18. $\lim_{h\to 0} \frac{(h+2)^3 - 8}{h}$

$\Rightarrow (h+2)^3 - 8$

$= (h^2+4h+4)(h+2) - 8$

$= h^2(h+2) + 4h(h+2) + 4(h+2) - 8$

$= h^3 + 2h^2 + 4h^2 + 8h + 4h + 8 - 8$

$= h^3 + 6h^2 + 12h$

$\Leftarrow \lim_{h\to 0} \frac{h^3 + 6h^2 + 12h}{h}$

$= \lim_{h\to 0} \frac{h(h^2 + 6h + 12)}{h}$

$= \lim_{h\to 0} h^2 + 6h + 12$

$\boxed{= 12}$

20. $\lim_{t\to 1} \frac{t^4 - 1}{t^3 - 1}$

$= \lim_{t\to 1} \frac{(t^2-1)(t^2+1)}{(t-1)(t^2+t+1)}$

$= \lim_{t\to 1} \frac{(t+1)(t-1)(t^2+1)}{(t-1)(t^2+t+1)}$

$= \lim_{t\to 1} \frac{(t+1)(t^2+1)}{t^2+t+1}$

$= \frac{(2)(2)}{3}$

$\boxed{= \frac{4}{3}}$ ■

1. $\lim_{x\to 2^-} \ln(4-x^2)$

$= \lim_{x\to 2^-} \ln(0^+)$

$\boxed{= \text{DNE}: -\infty}$

2. a. $\lim_{x\to 1} \dfrac{\frac{1}{(x-4)^2} - \frac{1}{x^2}}{x^2-4}$

$= \dfrac{\frac{1}{(1-4)^2} - 1}{-3}$

$= \dfrac{\frac{1}{9} - 1}{-3}$

$= \dfrac{-8/9}{-3}$

$\boxed{= \dfrac{8}{27}}$ ∎

b. $\lim_{x\to 2} \dfrac{\frac{1}{(x-4)^2} - \frac{1}{x^2}}{x^2-4}$

$= \lim_{x\to 2} \dfrac{\frac{x^2-(x-4)^2}{x^2(x-4)^2}}{x^2-4}$

$= \lim_{x\to 2} \dfrac{x^2-(x-4)^2}{(x^2-4)x^2(x-4)^2}$

$= \lim_{x\to 2} \dfrac{x^2-(x^2-8x+16)}{(x+2)(x-2)x^2(x-4)^2}$

$= \lim_{x\to 2} \dfrac{8x-16}{(x+2)(x-2)x^2(x-4)^2}$

$= \lim_{x\to 2} \dfrac{8(x-2)}{(x+2)(x-2)x^2(x-4)^2}$

$= \lim_{x\to 2} \dfrac{8}{(x+2)x^2(x-4)^2}$

$= \dfrac{8}{(4)(4)(-2)^2}$

$= \dfrac{8}{64} = \boxed{\dfrac{1}{8}}$ ∎

2. c. $\lim_{x\to 0^+} \dfrac{\frac{1}{(x-4)^2} - \frac{1}{x^2}}{x^2-4}$

$= \lim_{x\to 0^+} \dfrac{8}{(x+2)x^2(x-4)^2}$

$= \dfrac{8}{0+} = \infty$

$\lim_{x\to 0^-} \dfrac{\frac{1}{(x-4)^2} - \frac{1}{x^2}}{x^2-4}$

$= \lim_{x\to 0^-} \dfrac{8}{(x+2)x^2(x-4)^2}$

$= \dfrac{8}{0+} = \infty$

$\boxed{= \infty}$ ∎