HW #3

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Math N1A: Sections 2.3 - 2.4

Section: 2.3

22. $\lim_{u\to 2} \frac{\sqrt{4u+1}-3}{u-2}$

$= \lim_{u\to 2} \frac{(\sqrt{4u+1}-3)(\sqrt{4u+1}+3)}{(u-2)(\sqrt{4u+1}+3)}$

$= \lim_{u\to 2} \frac{4u+1-9}{(u-2)(\sqrt{4u+1}+3)}$

$= \lim_{u\to 2} \frac{4u-8}{(u-2)(\sqrt{4u+1}+3)}$

$= \lim_{u\to 2} \frac{4(u-2)}{(u-2)(\sqrt{4u+1}+3)}$

$= \lim_{u\to 2} \frac{4}{\sqrt{4u+1}+3}$

$= \frac{4}{6} = \boxed{\frac{2}{3}}$ ■

26. $\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right)$

$= \lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t(t+1)}\right)$

$= \lim_{t\to 0} \left(\frac{t+1}{t(t+1)} - \frac{1}{t(t+1)}\right)$

$= \lim_{t\to 0} \left(\frac{t+1-1}{t(t+1)}\right)$

$= \lim_{t\to 0} \left(\frac{t}{t(t+1)}\right)$

$= \lim_{t\to 0} \frac{1}{t+1}$

$\boxed{=1}$ ■

28. $\lim_{x\to 2} \frac{x^2-4x+4}{x^4-3x^2-4}$

$= \lim_{x\to 2} \frac{(x-2)^2}{(x^2-4)(x^2+1)}$

$= \lim_{x\to 2} \frac{(x-2)^2}{(x+2)(x-2)(x^2+1)}$

$= \lim_{x\to 2} \frac{(x-2)}{(x+2)(x^2+1)}$

$\boxed{=0}$ ■

38. $2x \le g(x) \le x^4 - x^2 + 2$

$\lim_{x\to 1} 2x \le \lim_{x\to 1} g(x) \le \lim_{x\to 1} x^4 - x^2 + 2$

$\lim_{x\to 1} 2x$ and $\lim_{x\to 1} x^4 - x^2 + 2$

$= 2$ $= 2$

Because $\lim_{x\to 1} 2x$ is 2, and $\lim_{x\to 1} x^4 - x^2 + 2$ is also 2 we can use squeeze thrm. to obtain that

$\boxed{\lim_{x\to 1} g(x) = 2}$

44. $\lim_{x\to -2} \frac{2-|x|}{2+x}$

$= \lim_{x\to -2} \frac{2--(x)}{2+x}$

$= \lim_{x\to -2} \frac{2+x}{2+x}$

$= \lim_{x\to -2} 1$

$\boxed{=1}$ ■

49. a. $\lim_{x\to 2} \frac{x^2+x-6}{|x-2|}$

$= \lim_{x\to 2} \frac{(x+3)(x-2)}{|x-2|}$

$|x-2| \begin{cases} x-2 \; ; \; x \ge 2 \\ -(x-2) \; ; \; x < 2 \end{cases}$

i. $= \lim_{x\to 2^+} \frac{(x+3)(x-2)}{x-2}$

$= \lim_{x\to 2^+} x+3$

$\boxed{=5}$ ■

ii. $\lim_{x\to 2^-} \frac{(x+3)(x-2)}{-(x-2)}$

$= \lim_{x\to 2^-} -(x+3)$

$= \lim_{x\to 2^-} -x-3$

$\boxed{=-5}$ ■

b. $\boxed{\text{Limit DNE}}$

65. $\lim_{x\to -2} \frac{3x^2+ax+a+3}{x^2+x-2}$

$= \lim_{x\to -2} \frac{3x^2+ax+a+3}{(x+2)(x-1)}$

Need to find how to get rid of $(x+2)$

Using synthetic division:

$$\begin{array}{r|ccc} -2 & 3 & a & (a+3) \\ & & -6 & -2(a-6) \\ \hline & 3 & (a-6) & (-a+15) \end{array}$$

Remainder

In order for $3x^2+ax+a+3$ to factor into $(x+2)(\cdots)$, $(-a+15)$ must equal 0.

$-a+15=0$

$\boxed{a=15}$

$\lim_{x\to -2} \frac{3x^2+15x+18}{x^2+x-2}$

$= \lim_{x\to -2} \frac{3(x^2+5x+6)}{(x+2)(x-1)}$

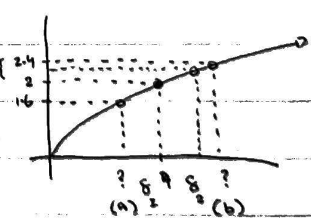
$= \lim_{x\to -2} \frac{3(x+3)(x+2)}{(x+2)(x-1)}$

$= \lim_{x\to -2} \frac{3(x+3)}{x-1}$

$\boxed{=-1}$

Section 2.4

3.



$\varepsilon = 0.4$

$\sqrt{a} = 1.6$ $\sqrt{b} = 2.4$

$a = 1.6^2$ $b = 2.4^2$

$a = 2.56$ $b = 5.76$

$\delta_1 = |4-2.56|$ $\delta_2 = |4-5.76|$

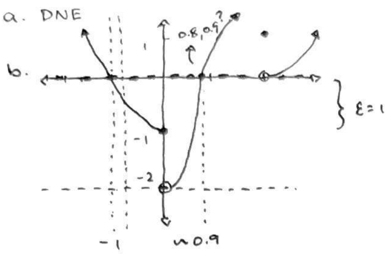
$= 1.44$ smaller $= 1.76$

$\boxed{1.44 > \delta}$

Luvreee's Problem

a. DNE

b.



$\boxed{\delta = 0.5}$ (Must be less than 0.9)

c. (b) does not show that $\lim_{x\to 0} g(x) = -1$ because it doesn't prove it. In the precise definition of a limit, it says that ε can be any number greater than 0. If you had set ε to 0.5, it wouldn't have worked. No value of δ would satisfy the ε window if ε = 0.5

Section 2.4

17. $\lim_{x\to -3}(1-4x) = 13$

Scratch:

If $0<|x+3|<\delta$, then $|(1-4x)-13|<\varepsilon$

$= |(1-4x)-13| < \varepsilon$

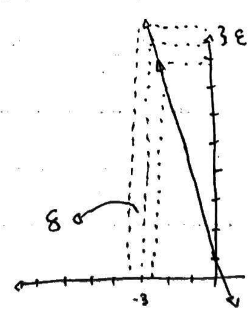
$= |-4x-12| < \varepsilon$

$= |-4(x+3)| < \varepsilon$

$= |-4||x+3| < \varepsilon$

$= 4|x+3| < \varepsilon$

$= |x+3| < \frac{\varepsilon}{4}$



Proof:

Given $\varepsilon > 0$, pick $\delta = \frac{\varepsilon}{4}$, if $0<|x+3|<\delta$, then $|(1-4x)-13|<\varepsilon$

$= |(1-4x)-13| < \varepsilon$

$= |-4x-12| < \varepsilon$

$= 4|x+3| < 4\delta$

$= 4|x+3| < 4\delta$

$= 4|x+3| < 4\left(\frac{\varepsilon}{4}\right)$

$= 4|x+3| < \varepsilon$ ∎

Thus $|1-4x-13| < \varepsilon$

$\therefore \lim_{-3}(1-4x) = 13$

30. $\lim_{x\to 2}(x^2+2x-7) = 1$

Scratch:

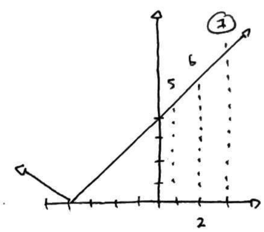
If $0<|x-2|<\delta$, then $|x^2+2x-7-1| < \varepsilon$

$= |x^2+2x-8| < \varepsilon$

$= |(x-2)(x+4)| < \varepsilon$

$= |x-2||x+4| < \varepsilon$

$= |x-2||x+4| < M|x-2| < \varepsilon$



$M = 7$

Assume:
$\delta \le 1$
$\delta < \frac{\varepsilon}{7}$

$= |x-2||x+4| < 7|x-2| < \varepsilon \;;\; |x-2| < \frac{\varepsilon}{7}$

$\therefore \delta = \min\left(1, \frac{\varepsilon}{7}\right)$

Proof:

Given $\varepsilon > 0$, pick $\delta = \min\left(1, \frac{\varepsilon}{7}\right)$

If $0<|x-2|<\delta$, then consider

$|x^2+2x-7-1| < \varepsilon$

$= |x-2||x+4| < |x+4|\delta$

$|x-2| < 1$

$-1 < x-2 < 1$

$1 < x < 3$

$5 < x+4 < 7$

$5 < |x+4| < 7$

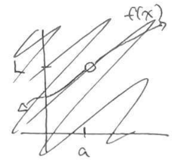
$\delta \le \frac{\varepsilon}{7}$

$|x+4|\delta < 7\delta$

$7\delta = 7\left(\frac{\varepsilon}{7}\right) = \varepsilon$

Thus $|x^2+2x-7-1| < \varepsilon$

Therefore $\lim_{x\to 2}(x^2+2x-7) = 1$ ∎

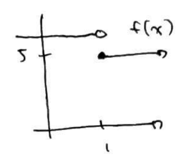


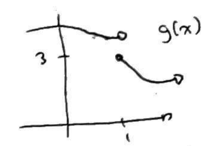
~~$\lim_{x\to a} f(x) = L$ (exists), but $f(a)$ does not exist. $\therefore$ False!~~

1. True. If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then $\lim_{x\to a}(f(x) + g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$.

Both exist!

2. False. Counter:





Both limits do not exist!

But $f(x) = 5$ and $g(x) = 3$

$\lim_{x\to 1} 5+3 = \boxed{8}$

3. $\lim_{x\to 0} \sin^2 x \sqrt{\ln(5+\sin(\frac{3}{x}))}$

$\Rightarrow -1 \le \sin(\frac{3}{x}) \le 1$

$\Rightarrow 4 \le \sin(\frac{3}{x}) + 5 \le 6$

$\Rightarrow \ln(4) \le \ln(\sin(\frac{3}{x}) + 5) \le \ln(6)$

$\Rightarrow \sqrt{\ln(4)} \le \sqrt{\ln(\sin(\frac{3}{x})+5)} \le \sqrt{\ln(6)}$

$\Rightarrow \sin^2 x \sqrt{\ln(4)} \le \sin^2 x \sqrt{\ln(\sin(\frac{3}{x})+5)} \le \sin^2 x \sqrt{\ln(6)}$

$\lim_{x\to 0} \sin^2 x \sqrt{\ln(4)}$ $\boxed{=0}$ $\qquad$ $\lim_{x\to 0} \sin^2 x \sqrt{\ln(6)}$ $\boxed{=0}$

$\therefore \lim_{x\to 0} \sin^2 x \sqrt{\ln(5+\sin(\frac{3}{x}))} = 0$ via Squeeze Thrm.

4. $\boxed{\text{False.}}$ The definition of the squeeze thrm. states that if $f(x) \le g(x) \le h(x)$ and if $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = \lim_{x\to a} h(x)$

In this scenario, the second condition, or $\lim_{x\to a} f(x) = \lim_{x\to a} h(x)$ is not met. Here is the work attempting to use squeeze thrm:

$\Rightarrow -1 \le \cos(\frac{3}{x-1}) \le 1$

$\Rightarrow e^{-1} \le e^{\cos(\frac{3}{x-1})} \le e^1$

$\Rightarrow \frac{1}{e} \le e^{\cos(\frac{3}{x-1})} \le e$

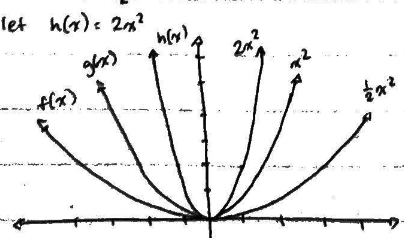
$\Rightarrow \frac{x^2}{e} \le e^{\cos(\frac{3}{x-1})} \le x^2 e$

$\lim_{x\to 1} \frac{x^2}{e} = \frac{1}{e} \ne \lim_{x\to 1} x^2 e = e$

Limits are not equal.

5. Let $f(x) = \frac{1}{2}x^2$

let $h(x) = 2x^2$



$f(x) \le g(x) \le h(x)$

$\lim_{x\to 0} \frac{1}{2}x^2 = 0$ $\qquad$ $\lim_{x\to 0} 2x^2 = 0$

$\therefore$ by sqz thrm, $\lim_{x\to 0} g(x) = 0$