HW #5

Math N1A: 2.6-3.2

Section 2.6

4. a. 2

b. -1

c. DNE: $-\infty$

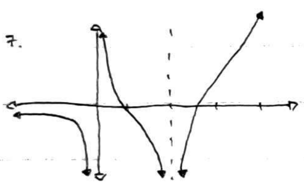
d. DNE: $-\infty$

e. DNE: ∞

f. $y=-1$, $y=2$

$x=0$, $x=2$

7.



16. $$\lim_{x\to\infty} \frac{-x^2+1}{x^3-x+1}$$

$$= \lim_{x\to\infty} \frac{x^2(-1+\frac{1}{x^2})}{x^3(1-\frac{1}{x^2}+\frac{1}{x^3})}$$

$$= \lim_{x\to\infty} \frac{-1+\frac{1}{x^2}}{x(1-\frac{1}{x^2}+\frac{1}{x^3})}$$

$$= \frac{-1}{\infty}$$

$$\boxed{=0}$$

21. $$\lim_{x\to\infty} \frac{(2x^2+1)^2}{(x-1)^2(x^2+x)}$$

$$= \lim_{x\to\infty} \frac{4x^4+4x^2+1}{x(x-1)^2(x+1)}$$

$$= \lim_{x\to\infty} \frac{4x^4+4x^2+1}{(x^3-x)(x-1)}$$

$$= \lim_{x\to\infty} \frac{4x^4+4x^2+1}{x^4-x^3-x^2+x}$$

$$= \lim_{x\to\infty} \frac{x^4(4+\frac{4}{x^2}+\frac{1}{x^4})}{x^4(1-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3})}$$

$$= \lim_{x\to\infty} \frac{4+\frac{4}{x^2}+\frac{1}{x^4}}{1-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}}$$

$$\boxed{=4}$$

23. $$\lim_{x\to\infty} \frac{\sqrt{1+4x^6}}{2-x^3}$$

$$= \lim_{x\to\infty} \frac{\sqrt{x^6(4+\frac{1}{x^6})}}{2-x^3}$$

$$= \lim_{x\to\infty} \frac{|x^3|\sqrt{4+\frac{1}{x^6}}}{x^3(\frac{2}{x^3}-1)}$$

$$= \lim_{x\to\infty} \frac{x^3\sqrt{4+\frac{1}{x^6}}}{x^3(\frac{2}{x^3}-1)}$$

$$= \lim_{x\to\infty} \frac{\sqrt{4+\frac{1}{x^6}}}{\frac{2}{x^3}-1}$$

$$= \frac{\sqrt{4}}{-1}$$

$$\boxed{=-2}$$

$$\sqrt{x^6} = |x^3| = \begin{cases} x^3 ; & x\ge 0 \\ -x^3 ; & x<0 \end{cases}$$

24. $$\lim_{x\to-\infty} \frac{\sqrt{1+4x^6}}{2-x^3}$$

$$= \lim_{x\to-\infty} \frac{\sqrt{x^6(4+\frac{1}{x^6})}}{2-x^3}$$

$$= \lim_{x\to-\infty} \frac{|x^3|\sqrt{4+\frac{1}{x^6}}}{x^3(\frac{2}{x^3}-1)}$$

$$= \lim_{x\to-\infty} \frac{-x^3\sqrt{4+\frac{1}{x^6}}}{x^3(\frac{2}{x^3}-1)}$$

$$= \lim_{x\to-\infty} \frac{-\sqrt{4+\frac{1}{x^6}}}{\frac{2}{x^3}-1}$$

$$= \frac{-\sqrt{4}}{-1}$$

$$\boxed{=2}$$

$$\sqrt{x^6} = |x^3| = \begin{cases} x^3 ; & x\ge 0 \\ -x^3 ; & x<0 \end{cases}$$

28. $$\lim_{x\to-\infty} \sqrt{4x^2+3x}+2x$$

$$= \lim_{x\to-\infty} \frac{4x^2+3x-4x^2}{\sqrt{4x^2+3x}-2x}$$

$$= \lim_{x\to-\infty} \frac{3x}{\sqrt{4x^2+3x}-2x}$$

$$= \lim_{x\to\infty} \frac{3x}{\sqrt{x^2(4+\frac{3}{x})}-2x}$$

$$= \lim_{x\to-\infty} \frac{3x}{|x|\sqrt{4+\frac{3}{x}}-2x}$$

$$= \lim_{x\to-\infty} \frac{3x}{-x\sqrt{4+\frac{3}{x}}-2x}$$

$$= \lim_{x\to-\infty} \frac{3x}{x(-\sqrt{4+\frac{3}{x}}-2)}$$

$$= \lim_{x\to-\infty} \frac{3}{-\sqrt{4+\frac{3}{x}}-2}$$

$$= \frac{3}{-4}$$

$$\boxed{=-\frac{3}{4}}$$

$$\sqrt{x^2} = |x| = \begin{cases} x , & x\ge 0 \\ -x ; & x<0 \end{cases}$$

38. $0 \le \sin^2 x \le 1$

$$\frac{0}{x^2+1} \le \frac{\sin^2 x}{x^2+1} \le \frac{1}{x^2+1}$$

$$\lim_{x\to\infty} \frac{0}{x^2+1} = 0 \qquad \lim_{x\to\infty} \frac{1}{x^2+1} = 0$$

$\therefore$ by squeeze, $\boxed{\lim_{x\to\infty} \frac{\sin^2 x}{x^2+1} = 0}$

Luurcet's problem

a. $\lim_{x\to 0^+} \frac{x}{\ln x}$; $\lim_{x\to 0^+} \ln x = -\infty$

$= \frac{0}{-\infty}$

$\boxed{= 0}$

b. $\lim_{x\to 1^-} \frac{x}{\ln x}$; $\lim_{x\to 1^-} \ln x = 0^-$

$= \frac{1}{0^-}$

$\boxed{= -\infty}$

Section 2.6

42. $\lim_{x\to\infty} [\ln(2+x) - \ln(1+x)]$

$= \lim_{x\to\infty} \ln\left(\frac{2+x}{1+x}\right)$

$= \lim_{x\to\infty} \ln\left(\frac{x(1+\frac{2}{x})}{x(1+\frac{1}{x})}\right)$

$= \lim_{x\to\infty} \ln\left(\frac{1+\frac{2}{x}}{1+\frac{1}{x}}\right)$

$= \ln(1)$

$\boxed{= 0}$

50. $\frac{1+x^4}{x^2-x^4}$

Vertical Asym:

$-x^4 + x^2 \neq 0$

$-x^2(x^2-1) \neq 0$

$-x^2(x+1)(x-1) \neq 0$

$\boxed{\therefore x \neq -1, 0, 1}$

Horizontal Asym:

$\lim_{x\to\infty} \frac{1+x^4}{x^2-x^4}$

$= \lim_{x\to\infty} \frac{x^4(1+\frac{1}{x^4})}{x^4(-1+\frac{1}{x^2})}$

$= \frac{1}{-1}$

$\boxed{= -1}$

$\lim_{x\to-\infty} \frac{1+x^4}{x^2-x^4}$

$= \lim_{x\to-\infty} \frac{x^4(1+\frac{1}{x^4})}{x^4(-1+\frac{1}{x^2})}$

$= \frac{1}{-1}$

$\boxed{= -1}$

$\boxed{\therefore y \neq -1}$

$\boxed{\text{Asymptotes: } x = -1, 0, 1 \quad y = -1}$

Section 2.7

18. a. $\frac{700-300}{60-20}$

$= \frac{400}{40}$

$\boxed{= 10}$

b. $[10, 50]$ c. $[40, 70]$

d. $\frac{200-400}{40-10}$

$\boxed{= -\frac{20}{3}}$

This value represents the average rate of change between $[10, 40]$, or the slope of the secant line between the points.

19. a. 20

b. Yes.

c. Yes. This can be determined because the graph slopes down at $f(70)$. However, $f'(60)$ only captures the slope at 60, and is not effected by what happens at $f(70)$. The downwards slope reduces the rate of change.

21. $f(2) = 4(2) - 5 = 3$

$f'(2) = \text{slope} = 4$

37. $f(x) = \sqrt{x}$

$a = 9$

42. $f(x) = \sin x$

$a = \frac{\pi}{6}$

Section 2.8

3. a. II

b. IV

c. I

d. III

24. $f(x) = -5x^2 + 8x + 4$; Domain: $\mathbb{R}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(-5(x+h)^2 + 8(x+h) + 4) - (-5x^2 + 8x + 4)}{h}$$

$$= \lim_{h\to 0} \frac{-5(x^2 + 2xh + h^2) + 8x + 8h + 4 + 5x^2 - 8x - 4}{h}$$

$$= \lim_{h\to 0} \frac{-5x^2 - 10xh - 5h^2 + 8x + 8h + 4 + 5x^2 - 8x - 4}{h}$$

$$= \lim_{h\to 0} \frac{-10xh - 5h^2 + 8h}{h}$$

$$= \lim_{h\to 0} \frac{(-10x - 5h + 8)h}{h}$$

$$= \lim_{h\to 0} -10x - 5h + 8$$

$$\boxed{= -10x + 8 \text{ ; Domain: } \mathbb{R}}$$

Section 3.1

6. $g(x) = \frac{7}{4}x^2 - 3x + 12$ (could've been done w/ shortcuts :/)

$$g'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(\frac{7}{4}(x+h)^2 - 3(x+h) + 12) - (\frac{7}{4}x^2 - 3x + 12)}{h}$$

$$= \lim_{h\to 0} \frac{(\frac{7}{4}(x^2 + 2xh + h^2) - 3x - 3h + 12) - \frac{7}{4}x^2 + 3x - 12}{h}$$

$$= \lim_{h\to 0} \frac{\frac{7}{4}x^2 + \frac{14}{4}xh + \frac{7}{4}h^2 - 3x - 3h + 12 - \frac{7}{4}x^2 + 3x - 12}{h}$$

$$= \lim_{h\to 0} \frac{\frac{14}{4}xh + \frac{7}{4}h^2 - 3h}{h}$$

$$= \lim_{h\to 0} \frac{14}{4}x + \frac{7}{4}h - 3$$

$$\boxed{= \frac{14}{4}x - 3}$$

Section 3.1

32. $f(x) = e^{x+1} + 1$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(e^{x+h+1} + 1) - (e^{x+1} + 1)}{h}$$

$$= \lim_{h\to 0} \frac{e^{x+h+1} + 1 - e^{x+1} - 1}{h}$$

$$= \lim_{h\to 0} \frac{e^{x+h+1} - e^{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{e^x e^h e - e^x e}{h}$$

$$= \lim_{h\to 0} e^x e \left(\frac{e^h - 1}{h}\right)$$

$$= e^x e \cdot \lim_{h\to 0} \left(\frac{e^h - 1}{h}\right)$$

$$= e^x e \cdot (1)$$

$$= e^x e$$

$$\boxed{= e^{x+1}}$$

35. $f(x) = x + \frac{2}{x} = x + 2x^{-1}$

$$f'(x) = 1 + 2(-1)x^{-2}$$

$$= 1 - 2x^{-2}$$

$$= 1 - \frac{2}{x^2}$$

$$f'(2) = 1 - \frac{2}{4} = \frac{1}{2}$$

$$\boxed{y - 3 = \frac{1}{2}(x - 2)}$$

Luuceet's Problems

A. $f(x) = 12x^7 - 5x^4 + 8\sqrt[4]{x} + 3 - \frac{6}{x^3} - \frac{7e^x}{2}$

$$f'(x) = 12(7)x^6 - 5(4)x^3 + 8(\tfrac{1}{4})x^{-\frac{3}{4}} + 0 + 6(3)x^{-4} - \frac{7}{2}e^x$$

$$\boxed{= 84x^6 - 20x^3 + 2x^{-\frac{3}{4}} + 18x^{-4} - \frac{7}{2}e^x}$$

B. $h(x) = \frac{(1 - 4\sqrt[3]{x})^2}{\sqrt{x}}$

$$= \frac{1 - 8\sqrt[3]{x} + 16x^{\frac{2}{3}}}{x^{\frac{1}{2}}}$$

$$= \frac{1}{\sqrt{x}} - 8x^{\frac{1}{3} - \frac{1}{2}} + 16x^{\frac{2}{3} - \frac{1}{2}}$$

$$= x^{-\frac{1}{2}} - 8x^{-\frac{1}{6}} + 16x^{\frac{1}{6}}$$

$$\boxed{h'(x) = -\frac{1}{2}x^{-\frac{3}{2}} + \frac{4}{3}x^{-\frac{7}{6}} + \frac{8}{3}x^{-\frac{5}{6}}}$$

46. $G(r) = \sqrt{r} + \sqrt[3]{r} = r^{\frac{1}{2}} + r^{\frac{1}{3}}$

$$\boxed{G'(r) = \frac{1}{2}r^{-\frac{1}{2}} + \frac{1}{3}r^{-\frac{2}{3}}}$$

$$\boxed{G''(r) = -\frac{1}{4}r^{-\frac{3}{2}} - \frac{2}{9}r^{-\frac{5}{3}}}$$

50. a. $p(t) = t^4 - 2t^3 + t^2 - t$

$v(t) = 4x^3 - 6x^2 + 2x - 1 = p'(t)$

$a(t) = 12x^2 - 12x + 2 = v'(t) = p''(t).$

$$\boxed{\begin{array}{l}\text{Velocity}: 4x^3 - 6x^2 + 2x - 1 \\ \text{Accel}: 12x^2 - 12x + 2\end{array}}$$

b. $12(1)^2 - 12(1) + 2$

$= 2$

$\boxed{2\ m/s^2}$

55. $y = 2x^3 + 3x^2 - 12x + 1$

$\frac{dy}{dx}\ 2x^3 + 3x^2 - 12x + 1$

$= 6x^2 + 6x - 12$

When tangent is horizonontal
oops.

$0 = 6x^2 + 6x - 12$

$0 = 6(x^2 + x - 2)$

$= 6(x+2)(x-1)$

$x = -2, 1$

$2(-2)^3 + 3(-2)^2 - 12(-2) + 1 = 21$

$2(1)^3 + 3(1)^2 - 12(1) + 1 = -6$

$\boxed{(-2, 21); (1, -6)}$

81. $mx+b$ must be tangent line to x^2 at 2.

$\frac{d}{dx} x^2 = 2x$ $\qquad (2)^2 = 4$

$m = 2(2)$ $\qquad$ Must intercept $(2, 4)$

$= 4$

$y - 4 = 4(x-2)$

$y = 4(x-2) + 4$

$y = 4x - 8 + 4$

$y = 4x - 4$; $b = -4$

$\boxed{m = 4,\ b = -4}$

Section: 3.2

10. $J(v) = (v^3 - 2v)(v^{-4} + v^{-2})$

$J'(v) = (v^3 - 2v)\frac{d}{dv}(v^{-4} + v^{-2}) + (v^{-4} + v^{-2})\frac{d}{dv}(v^3 - 2v)$

$$\boxed{= (v^3 - 2v)(-4v^{-5} - 2v^{-3}) + (v^{-4} + v^{-2})(3v^2 - 2)}$$

15. $y = \frac{t^3 + 3t}{t^2 - 4t + 3}$

$$\frac{dy}{dt} = \frac{(t^2 - 4t + 3)\frac{d}{dt}(t^3 + 3t) - (t^3 + 3t)\frac{d}{dt}(t^2 - 4t + 3)}{(t^2 - 4t + 3)^2}$$

$$\boxed{= \frac{(t^2 - 4t + 3)(3t^2 + 3) - (t^3 + 3t)(2t - 4)}{(t^2 - 4t + 3)^2}}$$

At $(2, y(2))$:

$$m = \frac{(2^2 - 4(2) + 3)(3(2)^2 + 3) - (2^3 + 3(2))(2(2) - 4)}{(2^2 - 4(2) + 3)}$$

$= -15$

$y(2) = \frac{2^3 + 3(2)}{2^2 - 4(2) + 3} = -14$; $(2, -14)$

$\boxed{y + 14 = -15(x - 2)}$

23. $f(x) = \dfrac{x^2 e^x}{x^2 + e^x}$

$$f'(x) = \frac{(x^2+e^x)\frac{d}{dx}(x^2e^x) - (x^2e^x)\frac{d}{dx}(x^2+e^x)}{(x^2+e^x)^2}$$

$$\boxed{= \frac{(x^2+e^x)(x^2e^x + e^x \cdot 2x) - (x^2e^x)(2x+e^x)}{(x^2+e^x)^2}}$$

30. $f(x) = \dfrac{x}{x^2-1}$

$$f'(x) = \frac{(x^2-1)\frac{d}{dx}(x) - x\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{(x^2-1) - x(2x)}{(x^2-1)^2}$$

$$\boxed{= -\frac{(x^2+1)}{(x^2-1)^2}}$$

$$\cancel{f''(x) = \frac{(x^2-1)^2\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x^2-1)^2}{(x^2-1)^4}}$$

$$\cancel{\frac{(x^2-1)^2(2x) - (x^2+1)(2x-2)}{(x^2-1)^4}}$$

$$f''(x) = \frac{(x^2-1)^2\frac{d}{dx}(-x^2-1) - (-x^2-1)\frac{d}{dx}\left[(x^2-1)^2\right]}{(x^2-1)^4}$$

$$\boxed{= \frac{(x^2-1)^2(-2x) - (-x^2-1)(4x^3-4x)}{(x^2-1)^4}}$$

$$\cancel{f'(x) = \frac{6}{2x-3}}$$

$$\cancel{\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}}$$

$$\cancel{= \lim_{h\to 0} \frac{\frac{6}{2(3+h)-3} - \frac{6}{2(3)-3}}{h}}$$

$$\cancel{= \lim_{h\to 0} \frac{\frac{6}{6+2h-3} - \frac{6}{3}}{h}}$$

$$\cancel{= \lim_{h\to 0} \frac{\frac{6}{6+2h-3} - 2}{h}}$$

$$\cancel{= \lim_{h\to 0} \frac{\frac{6-2(6+2h-3)}{6+2h-3}}{h}}$$

$$\cancel{\lim_{h\to 0} \frac{6-2(6+2h-3)}{h(6+2h-3)}}$$

$$\cancel{= \lim_{h\to 0} \frac{6-12-4h+6}{h(6+2h-3)}}$$

$$\cancel{\lim_{h\to 0} \frac{-4h}{h(6+2h-3)}}$$

$$\cancel{= \lim_{h\to 0} \frac{-4}{6+2h+(-3)}}$$

$$\cancel{m = -\frac{4}{3}}$$

$$\cancel{f(3) = \frac{6}{2(3)-3}} = \frac{6}{3} = 2$$

$$\cancel{y-2 = -\frac{4}{3}(x-3)}$$

~~50. a. P'(2) = F(2)G'(2) + G(2)F'(2)~~

2. $g(x) = \sqrt{1-4x}$

50. a. $P'(2) = F(2)G'(2) + G(2)F'(2)$

$$= (3)\left(\frac{1}{2}\right) + 2(0)$$

$$\boxed{= \frac{3}{2}}$$

b. $Q'(7) = \dfrac{G(7)F'(7) - F(7)G'(7)}{G(7)^2}$

$$= \frac{(1)\left(\frac{1}{4}\right) - (5)\left(-\frac{2}{3}\right)}{1}$$

$$= \frac{1}{4} + \frac{10}{3}$$

$$\boxed{= \frac{43}{12}}$$

1. $\frac{6}{2x-3} = f(x)$

$$\lim_{h\to 0}\left\{ \begin{aligned} f'(x) &= \frac{f(x+h)-f(x)}{h} \\ &= \frac{\frac{6}{2(x+h)-3} - \frac{6}{2x-3}}{h} \\ &= \frac{6(2x-3) - 6(2x+2h-3)}{h(2(x+h)-3)(2x-3)} \\ &= \frac{12x-18-12x-12h+18}{h(2(x+h)-3)(2x-3)} \\ &= \frac{-12h}{h(2(x+h)-3)(2x-3)} \\ &= \frac{-12}{(2(x+h)-3)(2x-3)} \end{aligned}\right.$$

$$= \frac{-12}{(2x-3)(2x-3)}$$

$$\boxed{= \frac{-12}{(2x-3)^2}}$$

$(3, f(3)) : \frac{-12}{(2(3)-3)^2} = -\frac{4}{3}$

$f(3) = 2$

$$\boxed{y-2 = -\frac{4}{3}(x-2)}$$

2. $g(x) = \sqrt{1-4x}$

$\cancel{g'(x) = \frac{f(x+h)-f(x)}{h}}$

$$g'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1-4(x+h)} - \sqrt{1-4x}}{h}$$

$$= \lim_{h\to 0} \frac{1-4x-4h-(1-4x)}{h(\sqrt{1-4(x+h)} + \sqrt{1-4x})}$$

$$= \lim_{h\to 0} \frac{1-4x-4h-1+4x}{h(\sqrt{1-4(x+h)} + \sqrt{1-4x})}$$

$$= \lim_{h\to 0} \frac{-4h}{h(\sqrt{1-4(x+h)} + \sqrt{1-4x})}$$

$$\cancel{\lim_{h\to 0} \frac{-4}{\sqrt{1-4x+4h}}}$$

$$= \lim_{h\to 0} \frac{-4}{\sqrt{1-4(x+h)} + \sqrt{1-4x}}$$

$$= \frac{-4}{\sqrt{1-4x} + \sqrt{1-4x}}$$

$$= \frac{-4}{2\sqrt{1-4x}}$$

$$\boxed{= \frac{-2}{\sqrt{1-4x}}}$$

$(-2, g(-2)) : \frac{-2}{\sqrt{1-4(-2)}} = -\frac{2}{3}$

$g(-2) = 3$

$$\boxed{y-3 = -\frac{2}{3}(x+2)}$$

~~3. $f(x) = \frac{e^x+4e^{-x}}{e^x - e^{-x}}$~~

$\cancel{= \frac{e^x\left(1+\frac{4e^{-x}}{e^x}\right)}{e^x\left(1-\frac{e^{-x}}{e^x}\right)}}$ ~~Factor from denom~~

$\cancel{\frac{1+\frac{4}{e^{2x}}}{1-\frac{1}{e^{2x}}}}$

$\cancel{\lim_{x\to -\infty} \frac{1+\frac{4}{e^{2x}}}{1-\frac{1}{e^{2x}}}}$

3. $f(x) = \frac{e^x + 4e^{-x}}{e^x - e^{-x}}$

e^{-x} is dom when $x \to -\infty$

$$\lim_{x\to -\infty} \frac{e^{-x}\left(4+\frac{e^x}{e^{-x}}\right)}{e^{-x}\left(-1+\frac{e^x}{e^{-x}}\right)}$$

$$\lim_{x\to -\infty} = \frac{4+e^{2x}}{-1+e^{2x}}$$

$$= \frac{4}{-1}$$

$$\boxed{= -4}$$

$$\lim_{x\to \infty} \frac{e^x\left(1+\frac{4e^{-x}}{e^x}\right)}{e^x\left(1-\frac{e^{-x}}{e^x}\right)}$$

$$= \lim_{x\to\infty} \frac{1+\frac{4}{e^{2x}}}{1-\frac{1}{e^{2x}}}$$

$$\boxed{= 1}$$