HW #6

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Math N1A: Sections 3.1 - 3.5

Section 3.1

64. a. $f(x) = x^2 + x$

$m = \frac{d}{dx} x^2 + x$

$= 2x + 1$

$y - y_1 = m(x - x_1)$

$f(x) + 3 = (2x+1)(x-2)$

$x^2 + x + 3 = 2x^2 - 4x + x - 2$

$0 = x^2 - 4x - 5$

$= (x-5)(x+1)$

$x = -1, 5$

$y+3 = (2(-1)+1)(x-2)$

$= -(x-2)$

$= -x + 2$

$\boxed{y = -x - 1}$

$y+3 = (2(5)+1)(x-2)$

$= 11(x-2)$

$= 11x - 22$

$\boxed{y = 11x - 25}$

20. Prove $f(x) = \cos x$, then $f'(x) = -\sin x$

$\lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$

$= \lim_{h\to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$

$= \lim_{h\to 0} \frac{\cos x \cos h - \cos x}{h} - \frac{\sin x \sin h}{h}$

$= \lim_{h\to 0} \frac{\cos x(\cos h - 1)}{h} - \frac{\sin x \sin h}{h}$

$= \lim_{h\to 0} \frac{\cos x}{h} \cdot \frac{\cos h - 1}{h} - \frac{\sin x \sin h}{h}$

$= \lim_{h\to 0} \frac{\cos x}{h} \cdot (0) - \frac{\sin x \sin h}{h}$

$= \lim_{h\to 0} - \frac{\sin x \sin h}{h}$

$= \lim_{h\to 0} -\sin x \left(\frac{\sin h}{h}\right)$

$= \lim_{h\to 0} -\sin x \cdot (1)$

$\boxed{= -\sin x}$

Section 3.3

14. $\frac{d}{dt}\left[\frac{\sin t}{1+\tan t}\right]$

$= \frac{(1+\tan t)\frac{d}{dt}\sin t - \sin t \frac{d}{dt}[1+\tan t]}{[1+\tan t]^2}$

$\boxed{= \frac{(1+\tan t)(\cos t) - (\sin t)(0 + \sec^2 x)}{[1+\tan t]^2}}$

16. $\frac{d}{dt} te^t \cot t$

$= \frac{d}{dt}[te^t][\cot t]$

$= te^t \frac{d}{dt}[\cot t] + \cot t \frac{d}{dt}[te^t]$

$= te^t(-\csc^2 t) + \cot t \left[t \frac{d}{dt} e^t + e^t \frac{d}{dt} t\right]$

$\boxed{= te^t(-\csc^2 t) + (\cot t)\left[t(te^{t-1}) + e^t\right]}$

22. $y = e^x \cos x$

$\frac{d}{dx} e^x \cos x$

$= e^x \frac{d}{dx}\cos x + \cos x \frac{d}{dx} e^x$

$= e^x(-\sin x) + \cos x\; e^x$

At $(0,1)$; $m = e^0(-\sin 0) + \cos(0)e^0$

$m = (1)(0) + (1)(1)$

$= 1$

$y - 1 = m(x - 0)$

$\boxed{y = x + 1}$

44. $\lim_{x \to 0} \frac{\sin 3x \sin 5x}{x^2}$

$= \lim_{x \to 0} \left[\frac{\sin 3x}{x} \cdot \frac{\sin 5x}{x} \right]$

$= \lim_{x \to 0} (3 \times 5)$

$\boxed{= 15}$

47. $\lim_{\theta \to 0} \frac{\cos\theta - 1}{2\theta^2}$

$= \lim_{\theta \to 0} \frac{(\cos\theta - 1)(\cos\theta + 1)}{2\theta^2(\cos\theta + 1)}$

$= \lim_{\theta \to 0} \frac{\cos^2\theta - 1}{2\theta^2(\cos\theta + 1)}$

$= \lim_{\theta \to 0} \frac{-\sin^2\theta}{2\theta^2(\cos\theta + 1)}$

$= \lim_{\theta \to 0} \left[\frac{\sin^2\theta}{\theta^2} \cdot -\frac{1}{2(\cos\theta + 1)} \right]$

$= \lim_{\theta \to 0} -\frac{1}{2(\cos\theta + 1)}$

$= -\frac{1}{2 \cdot 2}$

$\boxed{= -\frac{1}{4}}$

Section 3.4

8. $(1 + x + x^2)^{99}$

$\frac{d}{dx}(1 + x + x^2)^{99}$

$= 99(1 + x + x^2)^{98} \frac{d}{dx}(1 + x + x^2)$

$\boxed{= 99(1 + x + x^2)^{98} \frac{d}{dx}(2x + 1)}$

12. $\cos^2\theta$

$\frac{d}{d\theta}\cos^2\theta$

$= 2\cos\theta \cdot \frac{d}{d\theta}\cos\theta$

$\boxed{= -2\cos\theta \sin\theta}$

18. $(x^2+1)^3(x^2+2)^6$

$\frac{d}{dx}(x^2+1)^3(x^2+2)^6$

$= (x^2+1)^3 \frac{d}{dx}(x^2+2)^6 + (x^2+2)^6 \frac{d}{dx}(x^2+1)^3$

$\boxed{= (x^2+1)(6(x+2)^5 \cdot 2x) + (x^2+2)^6(3(x^2+1) \cdot 2x)}$

21. $\sqrt{\frac{x}{x+1}}$

$\frac{d}{dx}\left(\frac{x}{x+1}\right)^{\frac{1}{2}}$

$= \frac{1}{2}\left(\frac{x}{x+1}\right)^{-\frac{1}{2}} \frac{d}{dx}\left[\frac{x}{x+1}\right]$

$= \frac{1}{2}\left(\frac{x}{x+1}\right)\left(\frac{(x+1)\frac{d}{dx}x - x\frac{d}{dx}x+1}{(x+1)^2}\right)$

$\boxed{= \frac{1}{2}\left(\frac{x}{x+1}\right)\left(\frac{(x+1) - x}{(x+1)^2}\right)}$

28. $e^{\frac{z}{z-1}}$

$\frac{d}{dz} e^{\frac{z}{z-1}}$

$= e^{\frac{z}{z-1}} \frac{d}{dz}\frac{z}{z-1}$

$= e^{\frac{z}{z-1}} \cdot \frac{(z-1)\frac{d}{dz}z - z\frac{d}{dz}z-1}{(z-1)^2}$

$\boxed{= e^{\frac{z}{z-1}} \cdot \frac{(z-1) - z}{(z-1)^2}}$

42. $\sqrt{x + \sqrt{x + \sqrt{x}}}$

$\frac{d}{dx}\left(x + (x + x^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$= \frac{1}{2}\left(x + (x + x^{\frac{1}{2}})^{\frac{1}{2}}\right)^{-\frac{1}{2}} \cdot \frac{d}{dx}\left(x + (x + x^{\frac{1}{2}})^{\frac{1}{2}}\right)$

$= \frac{1}{2}\left(x + \sqrt{x + \sqrt{x}}\right)^{-\frac{1}{2}}\left(1 + \frac{d}{dx}\left[(x + \sqrt{x})^{\frac{1}{2}}\right]\right)$

$= \frac{1}{2}\left(x + \sqrt{x + \sqrt{x}}\right)^{-\frac{1}{2}}\left(1 + \frac{1}{2}(x + \sqrt{x})^{-\frac{1}{2}} \frac{d}{dx}(x + \sqrt{x})\right)$

$\boxed{= \frac{1}{2}\left(x + \sqrt{x + \sqrt{x}}\right)^{-\frac{1}{2}}\left(1 + \frac{1}{2}(x + \sqrt{x})^{-\frac{1}{2}}\left(1 + \frac{1}{2}x^{-\frac{1}{2}}\right)\right)}$

45. $y = \cos\sqrt{\sin(\tan \pi x)}$

$$\frac{d}{dx}\cos\left[\left(\sin(\tan \pi x)\right)^{\frac{1}{2}}\right]$$

$$= -\sin\left[\sqrt{\sin(\tan \pi x)}\right]\cdot\frac{d}{dx}\left[\sqrt{\sin(\tan \pi x)}\right]$$

$$= -\sin\left(\sqrt{\sin(\tan \pi x)}\right)\cdot\frac{1}{2}\left(\sin(\tan \pi x)\right)^{-\frac{1}{2}}\cdot\frac{d}{dx}\left[\sin(\tan \pi x)\right]$$

$$= -\sin\sqrt{\sin(\tan \pi x)}\cdot\frac{1}{2}\left(\sin(\tan \pi x)\right)^{-\frac{1}{2}}\cdot\cos(\tan \pi x)\cdot\frac{d}{dx}\tan \pi x$$

$$\boxed{= -\sin\sqrt{\sin(\tan \pi x)}\cdot\frac{1}{2}\left(\sin(\tan \pi x)\right)^{-\frac{1}{2}}\cdot\cos(\tan \pi x)\cdot\sec^2 \pi x\cdot\pi}$$ ■

49. $y = \sqrt{1-\sec t}$

$$y' = \frac{d}{dt}\sqrt{1-\sec t}$$

$$= \frac{1}{2}(1-\sec t)^{-\frac{1}{2}}\frac{d}{dt}\left[1-\sec t\right]$$

$$\boxed{= \frac{1}{2}(1-\sec t)^{-\frac{1}{2}}(-\sec t\tan t)}$$ ■

$$y'' = \frac{d}{dt}\left[\frac{1}{2}(1-\sec t)^{-\frac{1}{2}}(-\sec t\tan t)\right]$$

$$= \frac{1}{2}(1-\sec t)^{-\frac{1}{2}}\frac{d}{dt}(-\sec t\tan t) - \sec t\tan t\frac{d}{dt}\left[\frac{1}{2}(1-\sec t)^{-\frac{1}{2}}\right]$$

$$= \frac{1}{2}(1-\sec t)^{-\frac{1}{2}}\left[-\sec t\sec^2 t + \tan t(-\sec x\tan x)\right] - \sec t\tan t\left[\frac{1}{2}\left(-\frac{1}{2}(1-\sec t)^{-\frac{3}{2}}\cdot\frac{d}{dt}\left[1-\sec t\right]\right)\right]$$

$$\boxed{= \frac{1}{2}(1-\sec t)^{-\frac{1}{2}}\left[-\sec t\sec^2 t + \tan^2 t(-\sec x)\right] - \sec t\tan t\left[\frac{1}{2}\left(-\frac{1}{2}(1-\sec t)^{-\frac{3}{2}}(-\sec t\tan t)\right)\right]}$$

66. a. ~ -1

b. ~ 2

Section 3.5

3\. a. $\sqrt{x}+\sqrt{y}=1$

$\sqrt{x}+\sqrt{y(x)}=1$

$\frac{d}{dx}\left[\sqrt{x}+\sqrt{y(x)}\right]=0$

$=\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}y^{-\frac{1}{2}}\cdot\frac{dy}{dx}$

$-\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2}y^{-\frac{1}{2}}\cdot\frac{dy}{dx}$

$\boxed{\frac{-\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{2}y^{-\frac{1}{2}}}=\frac{dy}{dx}}$

b. $\sqrt{x}+\sqrt{y}=1$

$\sqrt{y}=-\sqrt{x}+1$

$y=(-\sqrt{x}+1)^2$

$y=-x-2\sqrt{x}+1$

$\frac{d}{dx}-x-2\sqrt{x}+1$

$\boxed{=-1-2\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}$

$-1-\frac{1}{\sqrt{x}}$

c. $\frac{-\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{2}y^{-\frac{1}{2}}}$

$=\frac{-x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$

$=-\frac{1}{\sqrt{x}}\cdot\frac{1}{\frac{1}{\sqrt{y}}}$

$=-\frac{\sqrt{y}}{\sqrt{x}}$

$=\frac{-(\sqrt{x}+1)}{\sqrt{x}}$

$=-\frac{\sqrt{x}+1}{\sqrt{x}}$

$=\left(\frac{\sqrt{x}}{\sqrt{x}}+\frac{1}{\sqrt{x}}\right)$

$=-\left(1+\frac{1}{\sqrt{x}}\right)$

$\boxed{=-1-\frac{1}{\sqrt{x}}}$ ✓

19\. $\sin xy=\cos(x+y)$

$\frac{d}{dx}\sin xy=\frac{d}{dx}\cos x\cos y-\sin x\sin y$

$\sin xy\cdot\frac{dy}{dx}=\left[\cos x(-\sin y)+\cos y(-\sin x)\right]-\left[\sin x\cos y+\sin y\cos x\right]$

$\boxed{\frac{dy}{dx}=\frac{\left[-\cos x\sin y+\cos y(-\sin x)\right]-\left[\sin x\cos y+\sin y\cos x\right]}{\sin xy}}$

31\. $2(x^2+y^2)^2=25(x^2-y^2)$

$\frac{d}{dx}2(x^2+y^2)^2=\frac{d}{dx}25(x^2-y^2)$

$4(x^2+y^2)(2x+2y\frac{dy}{dx})=25(2x-2y\frac{dy}{dx})$

$(4x^2+4y^2)(2x+2y\frac{dy}{dx})=50x-50y\frac{dy}{dx}$

$8x^3+8x^2y\frac{dy}{dx}+8y^2x+8y^3\frac{dy}{dx}=50x-50y\frac{dy}{dx}$

$8x^2y\frac{dy}{dx}+8y^3\frac{dy}{dx}+50\frac{dy}{dx}=-8x^3-8y^2x+50x$

$\frac{dy}{dx}(8x^2y+8y^3+50)=-8x^3-8y^2x+50x$

$\frac{dy}{dx}=\frac{-8x^3-8y^2x+50x}{8x^2y+8y^3+50}$

Plug in (3,1)

$=\frac{-8(3)^3-8(3)+50}{8(3)+8(3)^3+50}$

$\boxed{=-\frac{9}{13}}$

43\. $-8x^3-8y^2x+50x=0$

$-8x^3-8\left(\frac{5\sqrt{16x^2+25}}{4}\right)x+50x=0$

$\boxed{x=\frac{\pm5\sqrt{3}}{4} \quad y=\pm\frac{5}{4}}$

$2(x^2+y^2)^2=25(x^2-y^2)$

$2(x^4+2x^4y^4+y^4)=25x^2-25y^2$

$2x^4+4x^4y^4+y^4=25x^2-25y^2$

$4x^4y^4+y^4+25y^2=25x^2-2x^4$

$y^2(4x^4y^2+y^2+25)=25x^2-2x^4$

$y(4x^4y^2+y^2+25)^{\frac{1}{2}}=\sqrt{25x^2-2x^4}$

worked on solving for y on scratch paper

$y=\pm\frac{\sqrt{5\sqrt{16x^2+25}-4x^2-25}}{}$