HW 7 [illegible]

Math N1A: Sections 3.6, 3.7, 3.9

Section 3.6

11. $F(t) = (\ln t)^2 \sin t$

$\frac{d}{dt} (\ln t)^2 \sin t$

$= (\ln t)^2 \frac{d}{dt} \sin t + \sin t \frac{d}{dt} (\ln t)^2$

$= (\ln t)^2 \cos t + \sin t \left(2 \ln t \cdot \frac{d}{dt} \ln t\right)$

$\boxed{= (\ln t)^2 \cos t + \frac{2 \sin t \ln t}{t}}$ $\blacksquare$

12. $h(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$

$\frac{d}{dx} \ln\left(x + \sqrt{x^2-1}\right)$

$= \frac{1}{x + \sqrt{x^2-1}} \cdot \frac{d}{dx}\left[x + \sqrt{x^2-1}\right]$

$= \frac{1}{x + \sqrt{x^2-1}} \left(1 + \frac{d}{dx}\sqrt{x^2-1}\right)$

$= \frac{1}{x + \sqrt{x^2-1}} \left(1 + \frac{1}{2}(x^2-1)^{-\frac{1}{2}} \cdot \frac{d}{dx}\left[x^2-1\right]\right)$

$\boxed{= \frac{1}{x + \sqrt{x^2-1}} \left(1 + \frac{1}{2}(x^2-1)^{-\frac{1}{2}} \cdot 2x\right)}$ $\blacksquare$

21. $y = \tan\left[\ln(ax+b)\right]$

$\frac{d}{dx} \tan\left[\ln(ax+b)\right]$

$= \sec^2(\ln(ax+b)) \cdot \frac{d}{dx}\left[\ln(ax+b)\right]$

$= \sec^2(\ln(ax+b)) \left(\frac{1}{ax+b}\right) \cdot \frac{d}{dx}\left[ax+b\right]$

$\boxed{= \sec^2(\ln(ax+b)) \left(\frac{1}{ax+b}\right) \cdot a}$ $\blacksquare$

30. $f(x) = \ln(\ln(\ln x))$

$\frac{d}{dx} \ln(\ln(\ln x))$

$= \frac{1}{\ln(\ln x)} \cdot \frac{d}{dx} \ln(\ln x)$

$= \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{d}{dx} \ln x$

$\boxed{= \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}}$

44. $y = x^{\cos x}$

$\ln(y) = \cos x \cdot \ln(x)$

$\frac{d}{dx} \ln(y) = \frac{d}{dx}\left[\cos x \cdot \ln(x)\right]$

$\frac{1}{y} \cdot \frac{dy}{dx} = \cos x \frac{d}{dx} \ln x + \ln x \frac{d}{dx} \cos x$

$\frac{1}{y} \cdot \frac{dy}{dx} = \cos x \cdot \frac{1}{x} + \ln x \cdot -\sin x$

$\frac{dy}{dx} = y\left[\cos x \cdot \frac{1}{x} + \ln x \cdot -\sin x\right]$

$\boxed{= x^{\cos x}\left[\cos x \cdot \frac{1}{x} + \ln x \cdot -\sin x\right]}$ $\blacksquare$

52. $x^y = y^x$

$\ln(x^y) = \ln(y^x)$

$y \cdot \ln x = x \cdot \ln y$

$y \cdot \frac{d}{dx} \ln x + \ln x \cdot \frac{d}{dx} y = x \cdot \frac{d}{dx} \ln y + \ln y \cdot \frac{d}{dx} x$

$y \cdot \frac{1}{x} + \ln x \cdot \frac{dy}{dx} = x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \ln y \cdot (1)$

$\frac{y}{x} + \ln x \cdot \frac{dy}{dx} = \frac{x}{y} \cdot \frac{dy}{dx} + \ln y$

$\frac{dy}{dx} \cdot \ln x - \frac{dy}{dx} \cdot \frac{x}{y} = \ln y - \frac{y}{x}$

$\frac{dy}{dx}\left(\ln x - \frac{x}{y}\right) = \ln y - \frac{y}{x}$

$\boxed{\frac{dy}{dx} = \frac{\ln y - \frac{y}{x}}{\ln x - \frac{x}{y}}}$

Section 3.7

4. a. $f(t) = t^2 e^{-t}$

$v(t) = \frac{d}{dt} t^2 e^{-t}$

$= t^2 \frac{d}{dt} e^{-t} + e^{-t} \frac{d}{dt} t^2$

$\boxed{= t^2 (e^{-t} \cdot -1) + e^{-t}(2t)}$

b. $v(1) = (1)^2 (e^{-(1)} \cdot -1) + e^{-(1)}(2(1))$

$= -\frac{1}{e} + \frac{1}{e} \cdot 2$

$\boxed{= \frac{1}{e} \text{ feet/second}}$

c. $0 = t^2(e^{-t} \cdot -1) + e^{-t}(2t)$

$= t^2\left(-\frac{1}{e^t}\right) + \frac{1}{e^t} \cdot 2t$

$= -\frac{t^2}{e^t} + \frac{2t}{e^t}$

$= t\left(-\frac{t}{e^t} + \frac{2}{e^t}\right)$

$t = 0 \qquad 0 = -\frac{t}{e^t} + \frac{2}{e^t}$

$t = 2$

$\boxed{t = 0, 2}$

d. $t^2(e^{-t} \cdot -1) + e^{-t}(2t) \geq 0$

Cutpoints : 0, 2

Interval	Sign
$(-\infty, 0)$	-
$(0, 2)$	+
$(2, \infty)$	-

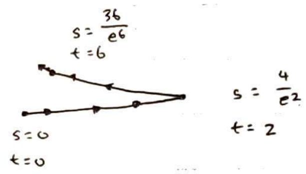
Particle is moving in the positive direction when $\boxed{t \in (0, 2)}$

e. Distance traveled from $(0,2)$: $|f(2) - f(0)| = \frac{4}{e^2}$

Distance traveled from $(2,6)$: $|f(6) - f(2)| = \frac{36}{e^6}$

$\Sigma D = \frac{4}{e^2} + \frac{36}{e^6} = \boxed{\frac{36 + 4e^4}{e^6}}$

f.



g. $\frac{d}{dt} t^2(-e^{-t}) + 2te^{-t}$

$= t^2(e^{-t}) - e^{-t}(2t) + 2t(-e^{-t}) + (e^{-t}) + (e^{-t})(2)$

When $t = 1$

$= e^{-1} - e^{-1}(2) + 2(-e^{-1}) + 2e^{-1}$

$= e^{-1} - 2e^{-1} - 2e^{-1} + 2e^{-1}$

$\boxed{= -e^{-1}}$

i. $t^2e^{-t} - 2te^{-t} - 2te^{-t} + 2e^{-t}$

$= t^2e^{-t} - 4te^{-t} + 2e^{-t}$

$= e^{-t}(t^2 - 4t + 2)$

Find cutpoints :

$0 = e^{-t}(t^2 - 4t + 2)$; e^{-t} is never 0, so it's "like" a constant

$= t^2 - 4t + 2$

$= \frac{4 \pm \sqrt{16 - 4(2)}}{2}$

$= 2 \pm \sqrt{2}$

Interval	Sign
$(-\infty, 2-\sqrt{2})$	+
$(2-\sqrt{2}, 2+\sqrt{2})$	-
$(2+\sqrt{2}, \infty)$	+

Speeding up : $t \in (0, 2-\sqrt{2}) \cup (2+\sqrt{2}, \infty)$

Slowing down : $t \in (2-\sqrt{2}, 2+\sqrt{2})$

8. a. $-16t^2+80t$

$-\frac{b}{2a} = \frac{80}{32} = \frac{5}{2}$

$-16\left(\frac{5}{2}\right)^2 + 80\left(\frac{5}{2}\right) = \boxed{100 \text{ feet}}$

b. $96 = -16t^2+80t$

$0 = -16t^2+80t-96$

$= 16t^2-80t+96$

$= 16(t^2-5t+6)$

$= 16(t-2)(t-3)$

$t = 2, 3$

$\frac{d}{dt} -16t^2+80t$

$= -32x+80$

$-32(2)+80 = 16$ $\quad$ $-32(3)+80 = -16$

$\boxed{\text{Velo. on way up: } 16 \text{ ft/s}}$ $\quad$ $\boxed{\text{Velo. on way down: } -16 \text{ ft/s}}$

Section 3.9

2. a. $A = \pi r^2$

$\frac{d}{dt}A = \frac{d}{dt}\pi r^2$

$\boxed{\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}}$

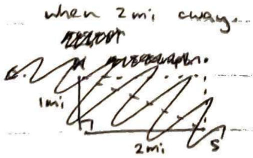
b. $\frac{dA}{dt} = 2\pi(30)\cdot(1)$

$\boxed{= 60\pi \ m^2/s}$

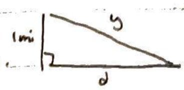
13. a. Altitude of 1mi

Spead of 500mi

b. Rate at which the dist. from plane to station is increasing when 2mi away.



c.



13. d. $y^2 = 1^2 + d^2$; $d = \sqrt{4-1} = \sqrt{3}$

e. $\frac{d}{dt}y^2 = \frac{d}{dt}1^2 + d^2$

$2y\frac{dy}{dt} = 2d\frac{dd}{dt}$

$\frac{dy}{dt} = \frac{2d}{2y}\cdot\frac{dd}{dt}$

$= \frac{\sqrt{3}}{2}\cdot\frac{dd}{dt}$

$\boxed{= \frac{500\sqrt{3}}{2} \text{ mph}}$

15. a. 15ft tall pole

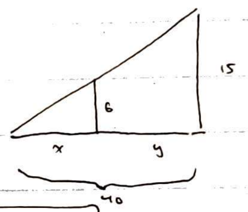
6ft tall man

Walks @ 5ft/s

40ft away

b. Tip of shadow

c.



d. $\frac{x}{6} = \frac{x+y}{15}$

$\cancel{e.\ 15x = 6(x+y)}$

$\cancel{15x = 6x+6y}$

$\cancel{9x = 6y}$

$\cancel{\frac{d}{dt}9x = \frac{d}{dt}6y}$

$\cancel{9\frac{dx}{dt} = 6\frac{dy}{dt}}$

$\cancel{9\frac{dx}{dt} = 6(5)}$

e. $15x = 6(x+y)$

$15x = 6x+6y$

$9x = 6y$

$3x = 2y$

$3\frac{dx}{dt} = 2\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{3}{2}\cdot\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{3}{2}(5)$

$= \frac{15}{2}$

Tip: $\frac{15}{2} + 5 = \boxed{\frac{25}{2} \text{ ft/s}}$

21. $A = \frac{1}{2}bh$; $100 = \frac{1}{2}bh \rightarrow 100 = \frac{1}{2}b(10)$

$b = 20$

$\frac{dA}{dt} = \frac{1}{2} \cdot \frac{d}{dt}(bh)$

$= \frac{1}{2}\left(b \cdot \frac{dh}{dt} + h \cdot \frac{db}{dt}\right)$

$2 = \frac{1}{2}\left(b \cdot \frac{dh}{dt} + h \cdot \frac{db}{dt}\right)$

$4 = 20 \cdot \frac{dh}{dt} + 10\frac{db}{dt}$

$4 = 20 + 10\frac{db}{dt}$

$-16 = 10\frac{db}{dt}$

$\frac{db}{dt} = -\frac{16}{10} = -\frac{8}{5}$

$\boxed{= -\frac{8}{5} \text{ cm/min}}$

1. False: