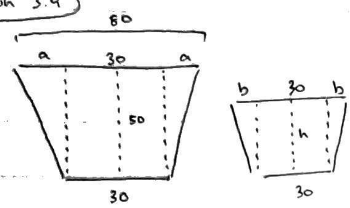
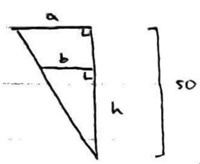
HW 8

Math N1A: Sections 3.9, 4.1, 4.2, 3.10

Section 3.9

27.





$80 - 2a = 30$

$a = 25$

$\frac{50}{25} = \frac{h}{b}$

$50b = 25h$

$b = \frac{1}{2}h$

$V = \frac{1}{2}(0.3 + [0.3 + 2b])\,10h$

$= \frac{1}{2}(0.6 + h)\,10h$

$= 10h\,(0.3 + \frac{1}{2}h)$

$= 3h + 5h^2$

$\frac{d}{dt}V = \frac{d}{dt}[3h + 5h^2]$

$\frac{dv}{dt} = 3\frac{dh}{dt} + 10h\frac{dh}{dt}$

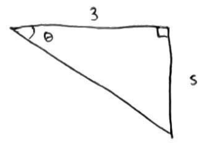
$\frac{dv}{dt} = 3\frac{dh}{dt} + 10(0.3)\frac{dh}{dt}$

$\frac{dv}{dt} = 6\frac{dh}{dt}$

$0.2 = 6\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = 1/30 \text{ m/min}}$

44.



$\frac{d\theta}{dt} = 4(2\pi) = 8\pi$

$\tan\theta = \frac{s}{3}$

$\tan\theta = \frac{1}{3}$

$\frac{d}{dt}\tan\theta = \frac{d}{dt}\left[\frac{1}{3}s\right]$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{3}\cdot\frac{ds}{dt}$

$3\sec^2\theta\cdot(8\pi) = \frac{ds}{dt}$

$3\left[\tan^2\theta + 1\right](8\pi) = \frac{ds}{dt}$

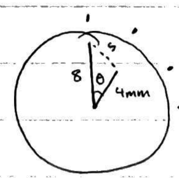
$\frac{ds}{dt} = 24\pi\left[\left(\frac{1}{3}\right)^2 + 1\right]$

$= 24\pi\left(\frac{10}{9}\right)$

$= \frac{240\pi}{9}$

$\boxed{= \frac{80\pi}{3} \text{ km/min}}$

50.



$\theta = \frac{1}{12}(2\pi) = \frac{\pi}{6}$

$s = \sqrt{8^2 + 4^2 - 2(8)(4)\cos\left(\frac{\pi}{6}\right)}$

$= \sqrt{80 - 32\sqrt{3}}$

$\frac{d\theta}{dt} = \frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$

$s^2 = 8^2 + 4^2 - 2(8)(4)\cos\theta$

$\frac{d}{dt}s^2 = \frac{d}{dt}\left[8^2 + 4^2 - 2(8)(4)\cos\theta\right]$

$2s\cdot\frac{ds}{dt} = 64\sin\theta\cdot\frac{d\theta}{dt}$

$\frac{ds}{dt} = \frac{64\sin\theta\cdot\frac{d\theta}{dt}}{2s}$

$\frac{ds}{dt} = \frac{32}{s}\cdot\sin\theta\,\frac{d\theta}{dt}$

$\frac{ds}{dt} = \frac{16}{s}\cdot\frac{d\theta}{dt}$

$\frac{ds}{dt} = \frac{16}{\sqrt{80 - 32\sqrt{3}}}\cdot\frac{d\theta}{dt}$

$\frac{ds}{dt} = \frac{16}{\sqrt{80 - 32\sqrt{3}}}\cdot\left(-\frac{11\pi}{6}\right)$

$\boxed{\frac{ds}{dt} = -\frac{176\pi}{\sqrt{80 - 32\sqrt{3}}} \text{ mm/hour}}$

Section 3.10

8. $L(x) = f(a) + f'(a)(x-a)$

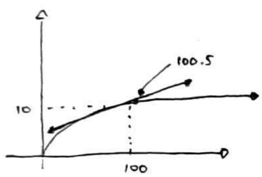
$f'(a) = \cos a$

$L(x) = \sin a + \cos a\,(x-a)$

$= \sin\frac{\pi}{6} + \cos\frac{\pi}{6}\left(x-\frac{\pi}{6}\right)$

$$\boxed{= \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x-\frac{\pi}{6}\right)}$$

26.



$f'(a) = \frac{1}{2}a^{-\frac{1}{2}}$

$L(x) = f(a) + f'(a)(x-a)$

$= 10 + \frac{1}{2}(100)^{-\frac{1}{2}}(x-100)$

$= 10 + \frac{1}{2}\cdot\frac{1}{10}(x-100)$

$= 10 + \frac{1}{20}(x-100)$

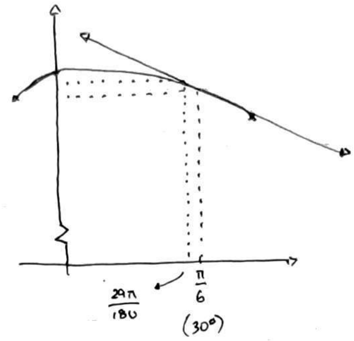
$L(100.5) = 10 + \frac{1}{20}(100.5-100)$

$= 10 + \frac{1}{40}$

$= 10.025$

$\sqrt{100.5}$ is a little bit less than 10.025

28.



$29° = 29\cdot\frac{\pi}{180}$ rads

$f'(a) = -\sin a$

$L(x) = f(a) + f'(a)(x-a)$

$= \cos\frac{\pi}{6} + \left(-\sin\frac{\pi}{6}\right)\left(x-\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2} + \left(-\frac{1}{2}\right)\left(x-\frac{\pi}{6}\right)$

$L\left(\frac{29\pi}{180}\right) = \frac{\sqrt{3}}{2} - \frac{1}{2}\left(\frac{29\pi}{180} - \frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2} - \frac{1}{2}\left(-\frac{\pi}{180}\right)$

$= \frac{\sqrt{3}}{2} + \frac{\pi}{360}$

$\cos 29°$ is a little bit less than $\frac{\sqrt{3}}{2} + \frac{\pi}{360}$

Section 4.1

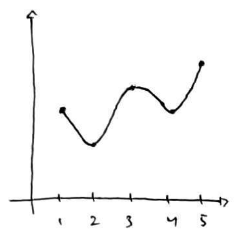
6. No abs. max

Local max: (3,4) ; (6,3)

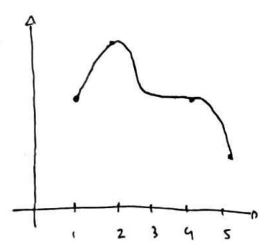
Local min : (2,2) ;

Abs Min : (4,1)

7.



10.



44. $f(x) = x^{-2} \ln x$

$f'(x) = \frac{d}{dx} x^{-2} \ln x$

$= x^{-2} \cdot \frac{d}{dx} \ln x + \ln x \cdot \frac{d}{dx} x^{-2}$

$= x^{-2} \cdot \frac{1}{x} + \ln x \cdot (-2x^{-3})$

$= \frac{1}{x^3} + \ln x \left(-2\frac{1}{x^3}\right)$

$= \frac{1}{x^3}\left(-2\ln x + 1\right)$

Domain: $x \in (0, \infty)$

$0 = \frac{1}{x^3}\left(-2\ln x + 1\right)$

$\frac{1}{x^3} = 0$

No Solution

$-2\ln x + 1 = 0$

$-2\ln x = -1$

$\ln x = \frac{1}{2}$

$\boxed{e^{\frac{1}{2}} = x}$

41. $f(\theta) = 2\cos\theta + \sin^2\theta$

$f'(\theta) = \frac{d}{d\theta}\left[2\cos\theta + \sin^2\theta\right]$

$= -2\sin\theta + 2\sin\theta \cdot \frac{d}{d\theta}\sin\theta$

$= 2\sin\theta\cos\theta - 2\sin\theta$

$= 2\sin\theta(\cos\theta - 1)$

function has no undefined points

$0 = 2\sin\theta(\cos\theta - 1)$

$2\sin\theta = 0$

$\sin\theta = 0$

$\theta = 0 + 2\pi k , \pi + 2\pi k$

$= \pi k$

$\cos\theta - 1 = 0$

$\cos\theta = 1$

$\theta = 2\pi k$

$\pi k , 2\pi k$

$\boxed{= \pi k}$

51. $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$

$f'(x) = \frac{d}{dx} 3x^4 - 4x^3 - 12x^2 + 1$

$= 12x^3 - 12x^2 - 24x$

$= 12x(x^2 - x - 2)$

$= 12x(x-2)(x+1)$

Function has no undef. pts.

$0 = 12x(x-2)(x+1)$

$x = -1, 0, 2$

Crit pts: $f(-1) = -4$ $\quad f(0) = 1$ $\quad f(2) = -31$

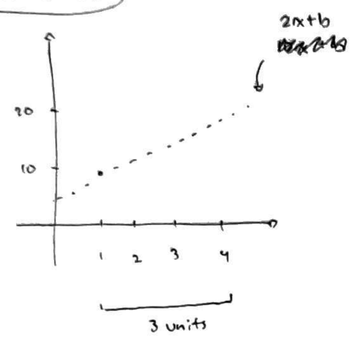
End pts: $f(-2) = 33$ $\quad f(3) = 28$

Absolute Max: (-2, 33)

Absolute Min (2, -31)

Section 4.2

25.



Minimum rise over 3 units : $2(3) = 6$

$10 + 6 = 16$

$\boxed{\text{Minimum value at } f(4) = 16}$

A. $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$= \dfrac{f(3) - f(2)}{3 - 2}$

$= \dfrac{0.5 - 1}{1}$

$= -\dfrac{1}{2}$

$-\dfrac{1}{2} = \dfrac{d}{dc}\left[\dfrac{1}{c-1}\right]$

$= \dfrac{d}{dc}(c-1)^{-1}$

$= -(c-1)^{-2}$ always negative

$\dfrac{1}{2} = \dfrac{1}{(c-1)^2}$

$2 = (c-1)^2$

$\sqrt{2} = c - 1$

$\boxed{c = \sqrt{2} + 1}$

B. $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$= \dfrac{f(2) - f(-1)}{2 + 1}$

$= \dfrac{1 + 0.5}{3}$

$= \dfrac{1.5}{3}$

However, as stated in A, $f'(x)$ must be negative. ∴ c cannot exist.

This does not disprove mean value thm, though, as $f(x)$ must be continuous. However, there is a vertical asymptote at $x = 1$. ∴ MVT holds.

~~Ans~~ Following Questions

1. $f(x) = x^{2/3}$ Domain: $\mathbb{R}$

$f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$

undefined at $f'(0)$

$g(x) = x^{-2/3}$

$= \frac{1}{x^{2/3}}$

Domain: $x \neq 0$, $\mathbb{R}$

$g'(x) = -\frac{2}{3}x^{-\frac{5}{3}}$

undefined at $g'(0)$

$x = 0$

No crit. numbers, 0 isn't within domain range

$\boxed{\text{False}}$

2. $f(x) = \sin x$

3. Proof: Atleast one root

$(x-20)^{101} + (x-20)^5 + x - 20 = e$

$f(x) = (x-20)^{101} + (x-20)^5 + x - 20 - e$

move e over, solve for 0, or a "root"

Because $f(x)$ is cont everywhere, as it is a polynomial, it is cont also between $[0, 50]$. By IVT, there is atleast a number $c \in (0, 50)$ such that $f(c) = 0$ exists.

$f(0) = (-20)^{101} + \dots$ (some small number less then 0)

$f(50) = (50-20)^{101} + \dots$ (some large number greater then 0)

Proof: Contradiction

Suppose for the sake of contradiction that there were atleast two roots. Call them a & b

$f(a) = 0$, $f(b) = 0$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{0}{b-a} = 0$$

$$101(x-20)^{100} \cdot \frac{d}{dx}[x-20] + 5(x-20)^4 \cdot \frac{d}{dx}[x-20] + 1$$

$$= 101(x-20)^{100} + 5(x-20)^4 + 1$$

Even degree polynomial + 1 ; never = 0

$\therefore$ By contradiction, $f(x)$ must only have one solution.