James Ding

Math N1A: Sections 4.1 – 4.9

Section 4.1

57. $f(t) = 2\cos t + \sin 2t$

$= 2\cos t + 2\sin t\cos t$

$= 2\cos t\,(1 + \sin t)$

$\frac{d}{dt} f(t) = 2\cos t(\cos t) + (1+\sin t)(-2\sin t)$

$= 2\cos^2 t - 2\sin t - 2\sin^2 t$

$0 = \frac{d}{dt} f(t)$

$= 2\cos^2 t - 2\sin t - 2\sin^2 t$

$= \cos^2 t - \sin t - \sin^2 t$

$= (1-\sin^2 t) - \sin t - \sin^2 t$

$= 1 - \sin t - 2\sin^2 t$

$= 2\sin^2 t + \sin t - 1$

$= (2\sin t - 1)(\sin t + 1)$

$0 = 2\sin t - 1$ $\quad$ $\sin t + 1 = 0$

$\sin t = \frac{1}{2}$ $\quad$ $\sin t = -1$

$t = \frac{\pi}{6} + 2\pi k,\ \frac{5\pi}{6} + 2\pi k$ $\quad$ $t = \frac{3\pi}{2} + 2\pi k$

When $f(t) = 0$ in $[0, \frac{\pi}{2}]$, $t = \frac{\pi}{6}$

$f(\frac{\pi}{6}) = \frac{3\sqrt{3}}{2} \approx 2.6$

$f(0) = 2$ $\quad$ $f(\frac{\pi}{2}) = 0$

$$\boxed{\begin{array}{l} \text{Abs. Max}: f(\frac{\pi}{6}) = \frac{3\sqrt{3}}{2} \\ \text{Abs. Min}: f(\frac{\pi}{2}) = 0 \end{array}}$$

Section 4.2

28. Because $f(x)$ and $g(x)$ are continuous on $[a,b]$ and differentiable on (a,b), if $h(x) = f(x) - g(x)$ MVT would apply for $h(x)$

$h'(c) = \frac{h(b) - h(a)}{b-a}$

Because $f(a) = g(a)$, that means $h(a) = 0$

Because $h'(c) = f'(c) - g'(c)$ and $g'(x) > f'(x)$, $h'(c) < 0$

contd

28. cont'd.

$\therefore \frac{h(b) - 0}{b-a} < 0$

Because $b > a$, $b - a$ is positive, which doesn't change the inequality of MVT being negative

$\therefore h(b) < 0$

$\boxed{\text{If } h(b) < 0, \text{ that means } g(b) > f(b)}$ ■

38. Let c be any number within the domain of $f(x)$

$f(x) = x$

$f'(c) = \frac{f(b) - f(a)}{b-a}$ $\quad$ $\boxed{\text{By contradiction, } f'(x) \neq 1}$

$= \frac{b-a}{b-a} = 1$ ✓ Not allowed.

c is the number, where it is a fixed point.

~~If there were other points however~~

Section 4.3

2. a. $(0,1) \cup (3,5) \cup (5,7)$

b. $(1,3)$

c. $(2,4) \cup (5,7)$

d. $(0,2) \cup (4,5)$

e. $(2,2)$; $(4,3)$

10. a. $\frac{d}{dx}\left[2x^3 - 9x^2 + 12x - 3\right]$

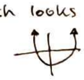
$= 6x^2 - 18x + 12$

$0 = 6x^2 - 18x + 12$

$= 6(x^2 - 3x + 2)$

$= 6(x-1)(x-2)$

follows form $a(x-p)(x-q)$ which looks like



$x = 1, 2$

Interval	Sign	Increasing / Decreasing	
$(-\infty, 1)$	+	Increasing	
			)- Local Max
$(1, 2)$	-	Decreasing	
			)- Local Min
$(2, \infty)$	+	Increasing	

b. With the previous chart

Local Max occurs when $x = 1$

$2(1)^3 - 9(1)^2 + 12(1) - 3 = \boxed{2}$

Local Min occurs when $x = 2$

$2(2)^3 - 9(2)^2 + 12(2) - 3 = \boxed{1}$

c. $\frac{d}{dx}\left[6x^2 - 18x + 12\right]$

$= 12x - 18$

$0 = 12x - 18$

$12x = 18$

$x = \frac{3}{2}$

Because $f''(x)$ follows the form $mx + b$, and m is positive

$$\boxed{\begin{array}{l}\text{Concave down: } (-\infty, \frac{3}{2}) \\ \text{Concave up: } (\frac{3}{2}, \infty)\end{array}}$$

Inflection when $x = \frac{3}{2}$

$2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + 12\left(\frac{3}{2}\right) - 3 = \frac{3}{2}$

$\boxed{\left(\frac{3}{2}, \frac{3}{2}\right)}$

13. a. $\frac{d}{dx}\left[\sin x + \cos x\right]$

$= \cos x - \sin x$

$0 = \cos x - \sin x$

$\sin x = \cos x$

$1 = \frac{\sin x}{\cos x} = \tan x$

$x = \frac{\pi}{4} + \pi k$

Points when $f'(x) = 0$ with $[0, 2\pi]$: $\frac{\pi}{4}, \frac{5\pi}{4}$

Interval	Sign	Increasing / Decreasing	
$(0, \frac{\pi}{4})$	+	Increasing	
			)- Local Max
$(\frac{\pi}{4}, \frac{5\pi}{4})$	-	Decreasing	
			)- Local Min
$(\frac{5\pi}{4}, 2\pi)$	+	Increasing	

b. on next page

13. b. With the previous chart

Local max occurs when $x = \frac{\pi}{4}$

$\sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \boxed{\sqrt{2}}$

Local Min occurs when $x = \frac{5\pi}{4}$

$\sin\frac{5\pi}{4} + \cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = \boxed{-\sqrt{2}}$

c. $\frac{d}{dx}\left[\cos x - \sin x\right]$

$= -\sin x - \cos x$

$0 = -\sin x - \cos x$

$\sin x = -\cos x$

$1 = -\tan x$

$x = \frac{3\pi}{4} + \pi k$

Points when $f''(x) = 0$ with $[0, 2\pi]$: $\frac{3\pi}{4}, \frac{7\pi}{4}$

Interval	Sign	Concavity
$(0, \frac{3\pi}{4})$	$-$	Concave down
$(\frac{3\pi}{4}, \frac{7\pi}{4})$	$+$	Concave up
$(\frac{7\pi}{4}, 2\pi)$	$-$	Concave down

} Inflection

} Inflection

Inflection when $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\sin\left(\frac{3\pi}{4}\right) + \cos\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0$

$\sin\left(\frac{7\pi}{4}\right) + \cos\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 0$

$\boxed{\left(\frac{3\pi}{4}, 0\right); \left(\frac{7\pi}{4}, 0\right)}$

~~[illegible]~~

~~[illegible]~~

Note $x \neq 1$

20. $f'(x) = \frac{d}{dx}\left[\frac{x^2}{x-1}\right]$ (First Derivative)

$= \frac{d}{dx}\left[(x-1)^{-1}(x^2)\right]$

$= (x-1)^{-1} \cdot \frac{d}{dx} x^2 + x^2 \cdot \frac{d}{dx}\left[(x-1)^{-1}\right]$

$= (x-1)^{-1} \cdot 2x + x^2 \cdot -(x-1)^{-2}$

$= \frac{2x}{x-1} + \frac{x^2}{-(x-1)^2}$

$= \frac{2x}{x-1} - \frac{x^2}{(x-1)^2}$

$= \frac{2x(x-1) - x^2}{(x-1)^2}$

$0 = 2x(x-1) - x^2$

$= 2x^2 - 2x - x^2$

$= x^2 - 2x$

$= x(x-2)$

$x = 0, 2$

$f''(x) = \frac{d}{dx}\left[\frac{2x(x-1) - x^2}{(x-1)^2}\right]$

$= \frac{d}{dx}\left[\frac{x(x-2)}{(x-1)^2}\right]$

$= \frac{d}{dx}\left[(x-1)^{-2}(x^2 - 2x)\right]$

$= (x-1)^{-2} \cdot \frac{d}{dx}(x^2 - 2x) + (x^2 - 2x) \cdot \frac{d}{dx}\left[(x-1)^{-2}\right]$

$= (x-1)^{-2}(2x-2) + (x^2 - 2x)\left(-2(x-1)^{-3}\right)$

$= \frac{2x-2}{(x-1)^2} + \frac{-2(x^2 - 2x)}{(x-1)^3}$

$= \frac{(2x-2)(x-1) + (-2)(x^2 - 2x)}{(x-1)^3}$

$= \frac{2}{(x-1)^3}$ ← cannot be 1

No zero points.

Interval	Sign	Increasing / Decreasing	
$(\infty, 0)$	+	Increasing	} Local Max
$(0, 1)$	−	Decreasing	} Nothing to see here :)
$(1, 2)$	−	Decreasing	} Local Min
$(2, \infty)$	+	Increasing	

Interval	Sign	Concavity
$(-\infty, 1)$	−	Concave down
$(1, \infty)$	+	Concave up

Local Max occurs when $x=0$

$\frac{0^2}{0-1} = \boxed{0}$

Local Min occurs when $x=2$

$\frac{2^2}{2-1} = \boxed{4}$

I prefer 1st derivative.

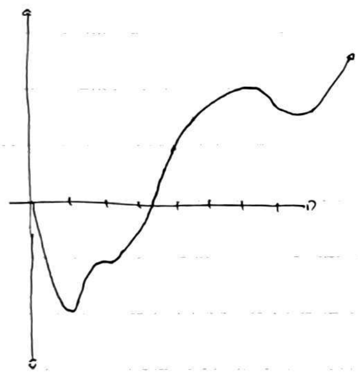
36. a. (1, 6), (8, 9) are increasing
(0, 1), (6, 8) are decreasing

b. Local Max: $x = 6$
Local Min: $x = 1, 8$

c. Concave up: (0, 2) ∪ (3, 5) ∪ (7, 9)
Concave down: (2, 3) ∪ (5, 7)

d. $x = 2, 3, 5, 7$

e.



Section 4.4

3. a. $\lim_{x\to a}\left[f(x) - p(x)\right]$

$= \lim_{x\to a} f(x) - \lim_{x\to a} p(x)$

$= 0 - \infty$

$\boxed{= -\infty}$

b. $\lim_{x\to a}\left[p(x) - q(x)\right]$

$= \lim_{x\to a} p(x) - \lim_{x\to a} q(x)$

$= \infty - \infty$

$\boxed{\text{Indet.}}$

c. $\lim_{x\to a}\left[p(x) + q(x)\right]$

$= \lim_{x\to a} p(x) + \lim_{x\to a} q(x)$

$= \infty + \infty$

$\boxed{= \infty}$

4. a. $\lim_{x\to a} f(x)^{\lim_{x\to a} g(x)}$

$= 0^0$

$\boxed{= \text{Indet}}$

b. $\lim_{x\to a} f(x)^{\lim_{x\to a} p(x)}$

$= 0^\infty$

$\boxed{= \text{Indet.}}$

c. $\lim_{x\to a}\left[h(x)\right]^{\lim_{x\to a} p(x)}$

$= 1^\infty$

$\boxed{= \text{Indet.}}$

d. $\lim_{x\to a} p(x)^{\lim_{x\to a} f(x)}$

$= \infty^0$

$\boxed{= \text{Indet.}}$

e. $\lim_{x\to a} p(x)^{\lim_{x\to a} q(x)}$

$= \infty^\infty$

$\boxed{= \infty}$

f. $\sqrt[\lim_{x\to a} q(x)]{\lim_{x\to a} p(x)}$

$\boxed{= \text{Indet.}}$

11. $\lim_{x\to 1} \frac{x^3 - 2x^2 + 1}{x^3 - 1} \Rightarrow \frac{0}{0}$

$\overset{LH}{=} \lim_{x\to 1} \frac{3x^2 - 4x + 0}{3x^2}$

$\boxed{= -\frac{1}{3}}$

41. $\lim_{x\to 0} \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4} \Rightarrow \frac{0}{6}$

$\overset{LH}{=} \lim_{x\to 0} \frac{-\sin x + x}{4x^3} \Rightarrow \frac{0}{0}$

$\overset{LH}{=} \lim_{x\to 0} \frac{-\cos x + 1}{12x^2} \Rightarrow \frac{0}{0}$

$\overset{LH}{=} \lim_{x\to 0} \frac{\sin x}{18x} \Rightarrow \frac{0}{0}$

$\overset{LH}{=} \lim_{x\to 0} \frac{\cos x}{24}$

$\boxed{= \frac{1}{24}}$

1. True. Because $a \neq b$, this denies the possibity of f being a constant, which has no absolute max. Therefore, it's true, as it denies the only counter.

2. False. Check x^3 at 0.

3. Must be differentiable too. Therefore False.

4. No. f(a) and f(b) do have to be part of [a,b] being cont, otherwise they can be random points which make no senso.