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Practice for Integral Exam

1. $\int_1^4 \frac{x^2 - x + 1}{\sqrt{x}}\, dx$

$= \int_1^4 \left(\frac{x^2}{\sqrt{x}} - \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right) dx$

$= \int_1^4 \left[x^{3/2} - x^{1/2} + x^{-1/2}\right] dx$

$= \left[\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + 2\sqrt{x}\right]\Big|_1^4 = \boxed{\frac{146}{15}}$

2. $\int \frac{\cos(\ln x)}{x}\, dx$

$\int \frac{\cos(\ln x)}{x}\, dx$

let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$= \int \cos u\, du$

$= \sin u + C$

$\boxed{= \sin(\ln x) + C}$

3. $\int \sin x \cos(\cos x)\, dx$

let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = -\frac{du}{\sin x}$

$= \int \cos u\, dx$

$= \sin u + C$

$\boxed{= \sin(\cos x) + C}$

4. $\int (1 - x)\sqrt{2x - x^2}\, dx$

let $u = 2x - x^2$

$\frac{du}{dx} = 2 - 2x$

$= 2(1-x)$

$dx = \frac{du}{2(1-x)}$

$= \int \frac{\sqrt{u}}{2}\, du$

$= \frac{1}{3}u^{3/2} + C$

$\boxed{= \frac{1}{3}(2x-x^2)^{3/2} + C}$

5. $\int_1^2 \frac{1}{2-3x}\,dx$

let $u = 2-3x$

$\frac{du}{dx} = -3$

$dx = -\frac{du}{3}$

$= \int \frac{1}{u} \cdot -\frac{du}{3}$

$= \frac{1}{3}\ln|u| \Big|_1^2$

$= -\frac{1}{3}\ln|2-3x| \Big|_1^2$

$= -\frac{1}{3}\ln|2-3x| \Big|_1^2$

$= -\frac{1}{3}\ln(3x-2) \Big|_1^2 = \boxed{-\frac{2\ln 2}{3}}$

$\cancel{= -\frac{1}{3}\ln(2-3x)\Big|_1^{2/3} + -\frac{1}{3}\ln(-2+3x)\Big|_{2/3}^{2}}$

$\cancel{= -\frac{1}{3}\ln(-2+3x)\Big|_0^{2/3} + -\frac{1}{3}\ln(2-3x)}$

6. $\int \frac{e^x}{(e^x+1)\ln(e^x+1)}\,dx$

let $u = e^x+1$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

$= \int \frac{1}{u\ln u}\,dx$

let $w = \ln u$

$\frac{dw}{du} = \frac{1}{u}$

$du = u\,dw$

$= \int \frac{1}{w}\,dw$

$= \ln w + c = \ln(\ln u) + c = \boxed{\ln(\ln(e^x+1)) + c}$

7. $\int_0^{\frac{\pi}{4}} (1+\tan t)^3 (\sec t)^2\,dt$

let $u = 1+\tan t$

$\frac{du}{dt} = \sec^2 t$

$dt = \frac{du}{\sec^2 t}$

$= \int u^3\,dt$

$= \frac{1}{4}u^4 \Big|_0^{\pi/4}$

$= \frac{1}{4}(1+\tan t)^4 \Big|_0^{\pi/4}$

$\boxed{= \frac{15}{4}}$

8. $\int_1^e \frac{\ln x}{x}\,dx$

let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$= \int_1^e u\,du$

$= \frac{1}{2}u^2 \Big|_1^e$

$= \frac{1}{2}(\ln x)^2 \Big|_1^e$

$\boxed{= \frac{1}{2}}$

9. $\int_1^4 \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx$

let $u = 1+\frac{1}{x}$

$\frac{du}{dx} = -\frac{1}{x^2}$

$dx = -x^2\,du$

$= \int_1^4 -\sqrt{u}\,du$

$= -\int_1^4 \sqrt{1+\frac{1}{x}}\,dx$

$\boxed{= \frac{4\sqrt{2}}{3} - \frac{5\sqrt{5}}{12}}$

10. $\int_0^4 \frac{1}{(x-2)^3}\,dx$

DNE. f(x) must be cont. within interval.

11. $\int_0^4 x^2(x^3+8)^2\,dx$

let $u = x^3+8$

$\frac{du}{dx} = 3x^2$

$dx = \frac{du}{3x^2}$

$\cancel{\int_0^4 x^2 (x^3+8)^2}$

$\int_0^4 \frac{1}{3}u^2\,dx$

$\frac{1}{9}u^3 \Big|_0^4$

$= \frac{1}{9}(x^3+8)^3 \Big|_0^4$

$\boxed{= \frac{372736}{9}}$

12. $\int_{-3}^2 x\sqrt{9-x^2}\,dx$

let $u = 9-x^2$

$\frac{du}{dx} = -2x$

$dx = -\frac{du}{2x}$

$= \int_{-3}^2 -\frac{\sqrt{u}}{2}\,du$

$= -\frac{1}{3}u^{3/2} \Big|_{-3}^2$

$\boxed{= -\frac{5^{3/2}}{3}}$

13. $\int_0^{\frac{\pi}{4}} \sin 4t\, dt$

$= ~~1861~~$

$-\frac{1}{4} \cos 4t \Big|_0^{\pi/4}$

$\boxed{= \frac{1}{2}}$

14. $\int \cot 8x$ dx

$\int \frac{\cos 8x}{\sin 8x} dx$

$=$ let $u = \sin 8x$

$\frac{du}{dx} = 8\cos x$

$dx = 8\frac{du}{\cos x}$

$\int \frac{1}{8u} du$

$\frac{1}{8} \ln u + C$

$\boxed{\frac{1}{8} \ln(\sin 8x) + C}$

15. Show: $\frac{1}{3}$ < ln 1.5 < $\frac{5}{12}$ by comparing areas.

$\ln\left(\frac{3}{2}\right)$

$\square < \ln 1.5 < \square$

$\frac{1}{3} < \ln 1.5 < \frac{1}{2}\left(\frac{1}{2}+\frac{1}{3}\right)$

$\frac{1}{3} < \ln 1.5 < \frac{5}{12}$ ✓

16. Find the derivative of y = $\int_0^{2x^2} e^{t^2}$ dt.

$e^{(2x^2)^2} \cdot 4x$

$\boxed{e^{4x^4} \cdot 4x}$