James Day

Average Velocity/ Instantaneous Velocity Practice

1. A rock breaks loose from the top of a tall cliff. Experiments show that a dense solid object dropped from rest to fall freely near the surface of the earth will fall $s(t) = 16t^2$ feet in the first t seconds.
   a. Find the average velocity of the rock for the first two seconds.

Find avg. velo $[0, 2]$ $\xrightarrow{\text{use rise/run}}$ $\dfrac{s(2) - s(0)}{2-0} = \dfrac{16(4)}{2} = \boxed{32 \text{ fps}}$

   b. Find the instantaneous velocity of the rock at t = 2.

let $f(x) = s(x)$

$$\lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \text{ ; let } a = 2$$

$$= \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h\to 0} \frac{16(h+2)^2 - 16(2)^2}{h}$$

$$= \lim_{h\to 0} \frac{16(h^2 + 4h + 4) - 16(4)}{h}$$

$$= \lim_{h\to 0} \frac{16h^2 + 64h + 64 - 64}{h}$$

$$= \lim_{h\to 0} \frac{16h^2 + 64h}{h}$$

$$= \lim_{h\to 0} \frac{h(16h + 64)}{h}$$

$$= \lim_{h\to 0} 16h + 64$$

$\boxed{= 64}$ fps

2. If a ball is thrown into the air with a velocity of 40ft/sec, its height in feet t sec later is given by $s(t) = 40t - 16t^2$.
   a) Find the average velocity for the time period beginning when t = 2 and lasting
      i) .5 seconds $\to [2, 2.5]$

$$\frac{s(2.5) - s(2)}{2.5 - 2} = \boxed{-32 \text{ fps}}$$

      ii) .1 seconds $\to [2, 2.1]$

$$\frac{s(2.1) - s(2)}{2.1 - 2} = \boxed{-25.6 \text{ fps}}$$

      iii) .05 seconds $\to [2, 2.05]$

$$\frac{s(2.05) - s(2)}{2.05 - 2} = \boxed{-24.8 \text{ fps}}$$

      iv) .01 seconds $\to [2, 2.01]$

$$\frac{s(2.01) - s(2)}{2.01 - 2} = \boxed{-24.16 \text{ fps}}$$

   b) Estimate the instantaneous velocity when t = 2.

$\boxed{-24 \text{ fps}}$

3. A tank holds 1000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume V of water remaining in the tank after t minutes.

t(min)	5	10	15	20	25	30
V(gal)	694	444	250	111	28	0

If P is the point (15, 250) on the graph of V, find the slopes of the secant lines PQ when Q is the point on the graph with t = 5, 10, 20, 25, and 30.

When $t=5$: $\frac{250-694}{15-5}$ $\boxed{=-44.4 \text{ gal/min}}$

when $t=10$: $\frac{250-444}{15-10}$ $\boxed{=-38.8 \text{ gal/min}}$

when $t=20$: $\frac{111-250}{20-15}$ $\boxed{=-27.8 \text{ gal/min}}$

when $t=25$: $\frac{28-250}{25-15}$ $\boxed{=-22.2 \text{ gal/min}}$

When $t=30$: $\frac{0-250}{30-15}$ $\boxed{=-16.667 \text{ gal/min}}$

4. Find the equation of the tangent line to the graph of $y = x^3 + 3x$ at $x = 1$.

Let $f(x) = x^3 + 3x$

$$m = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \text{ ; let } a=1$$

$$= \lim_{h\to 0} \frac{(a+h)^3 + 3(a+h) - [a^3+3a]}{h}$$

$$= \lim_{h\to 0} \frac{(h+1)^3 + 3(h+1) - 4}{h}$$

$$= \lim_{h\to 0} \frac{h^3+3h^2+3h+1 + 3h+3 - 4}{h}$$

$$= \lim_{h\to 0} \frac{h^3+3h^2+6h}{h}$$

$$= \lim_{h\to 0} \frac{h(h^2+3h+6)}{h}$$

$$= \lim_{h\to 0} h^2+3h+6$$

$$m \boxed{=6}$$

Slope

When $x=1$, $f(x) = (1)^3 + 3(1)$
$= 4$

$$\boxed{y-4 = 6(x-1)}$$