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Date: ____________

First day take-home quiz

In the sport of basketball, when a player is "fouled," they have the opportunity to take two "free-throws," where the player shoots the ball two times and can give his team one point for each basket made.

An important statistic in basketball is a player's free-throw success percentage over the basketball season. For example, if a player has had 10 free-throw attempts and made 8 baskets, their percentage is 80%.

Interestingly, the percentage is always rounded to the nearest percent (for the ease of the viewing public, which probably does not know math ☺). For example, if a player has made 7 out of 22 free-throws, then the reported percentage is 32%.

During a game, the announcer says that a player is making 78% of their free throws at the moment that the player is fouled. The player makes two free-throw attempts and makes the first one but misses the second attempt. After the statistics are updated, the player is now making 76% of their free-throws.

For this player, what are all possible numbers of free-throws made and attempted so far this season (including the ones that were just made)? (Hint: There are a finite number of solutions)

Take Home Quiz 1

AP Calc BC

Terms

n_0: The initial number of rounds

S_0: The initial number of successes

Requirements

$$\frac{775}{1000} \leq \frac{s}{n} < \frac{785}{1000} \quad \text{AND} \quad \frac{755}{1000} \leq \frac{s+1}{n+2} < \frac{765}{1000}$$

What numbers for s and n s.t these conditions are met?

Inequalities (in terms of n)

$$\frac{775}{1000} n \leq s < \frac{785}{1000} n$$

$$n \leq \frac{1000}{775} s \quad \text{AND} \quad n > \frac{1000}{785} s$$

$$\frac{755}{1000}(n+2) \leq s+1 < \frac{765}{1000}(n+2)$$

$$\frac{755}{1000} n + \frac{151}{100} \leq s+1 < \frac{765}{1000} n + \frac{153}{100}$$

$$\frac{755}{1000} n \leq s - \frac{51}{100} \quad \text{AND} \quad \frac{765}{1000} n > s - \frac{53}{100}$$

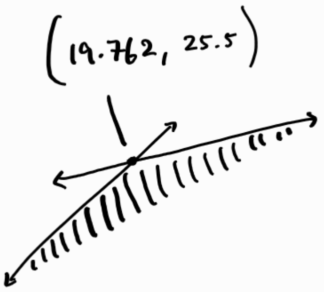
$$n \leq \frac{1000}{755} s - \frac{102}{151} \quad \text{AND} \quad n > \frac{1000}{765} s - \frac{106}{153}$$

Intersections

$$n \leq \frac{1000}{775} s \quad \text{AND} \quad n \leq \frac{1000}{755} s - \frac{102}{151}$$

$$\frac{1000}{775} s = \frac{1000}{755} s - \frac{102}{151}$$

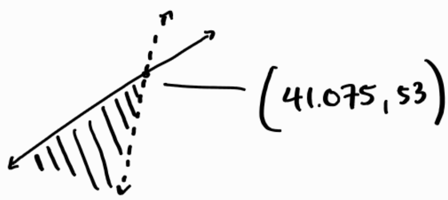
$$s = \frac{1581}{80} \approx 19.762 \qquad n = \frac{51}{2} = 25.5$$



$$n \leq \frac{1000}{775} s \quad \text{AND} \quad n > \frac{1000}{765} s - \frac{106}{153}$$

$$\frac{1000}{775} s = \frac{1000}{765} s - \frac{106}{153}$$

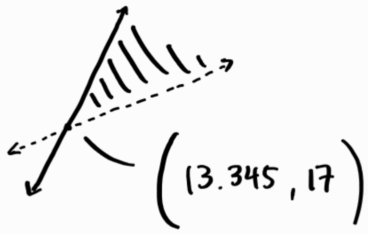
$$S_0 = \frac{1643}{40} = 41.075 \qquad n_0 = 53$$



$$n > \frac{1000}{785}s \quad \text{AND} \quad n \leq \frac{1000}{755}s - \frac{102}{151}$$

$$\frac{1000}{785}s = \frac{1000}{755}s - \frac{102}{151}$$

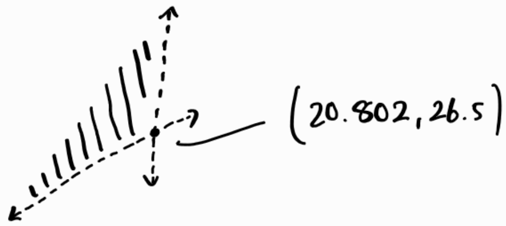
$$s = \frac{2669}{200} = 13.345 \qquad n = 17$$



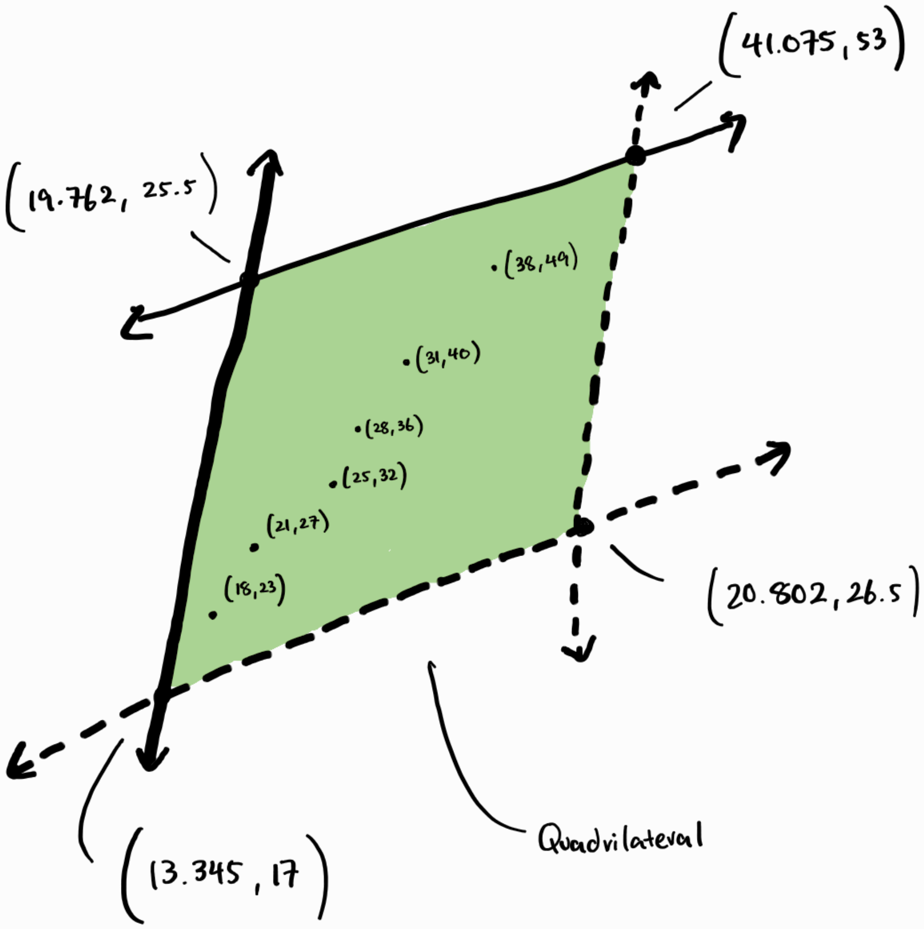
$$n > \frac{1000}{785}s \quad \text{AND} \quad n > \frac{1000}{765}s - \frac{106}{153}$$

$$\frac{1000}{785}s = \frac{1000}{765}s - \frac{106}{153}$$

$$s = \frac{8321}{400} \approx 20.802 \qquad n = \frac{53}{2} = 26.5$$



Combining Inequalities



Calculating Solutions

Formally, the solutions for this problem can be defined in terms of s, n

$$\frac{775}{1000} \leq \frac{s}{n} < \frac{785}{1000} \quad \text{AND} \quad \frac{755}{1000} \leq \frac{s+1}{n+2} < \frac{765}{1000} \quad \text{WHERE } s \text{ and } n \text{ are whole numbers}$$

Intuitively, the solutions for this problem are points (s, n) in the quadrilateral that are whole numbers.

From this, know that our solutions must lie within these bounds:

$$s \in [14, 41] \quad \text{AND} \quad n \in (17, 53)$$

Given the bounds, we can iterate through all the possible combinations (manually, or with a computer)

This work can be converted into psuedocode / logic

```
FOR X IN 14...41
    FOR Y IN 17...53
        IF (X,Y) SATISFIES INEQUALITIES
            ADD (X,Y) TO SOLUTIONS
```

You can technically make this more effecient with flooring / ceiling the max/min values of the quadrilateral at X

After implementing this, you get the following solutions:

$(18, 25)$; $(21, 27)$; $(25, 32)$; $(28, 36)$; $(31, 40)$; $(38, 49)$

Therefore, the final number of successes and attempts are:

$$\boxed{(19, 27);\ (22, 29);\ (26, 34);\ (29, 38);\ (32, 42);\ (39, 51)}$$

Using Desmos

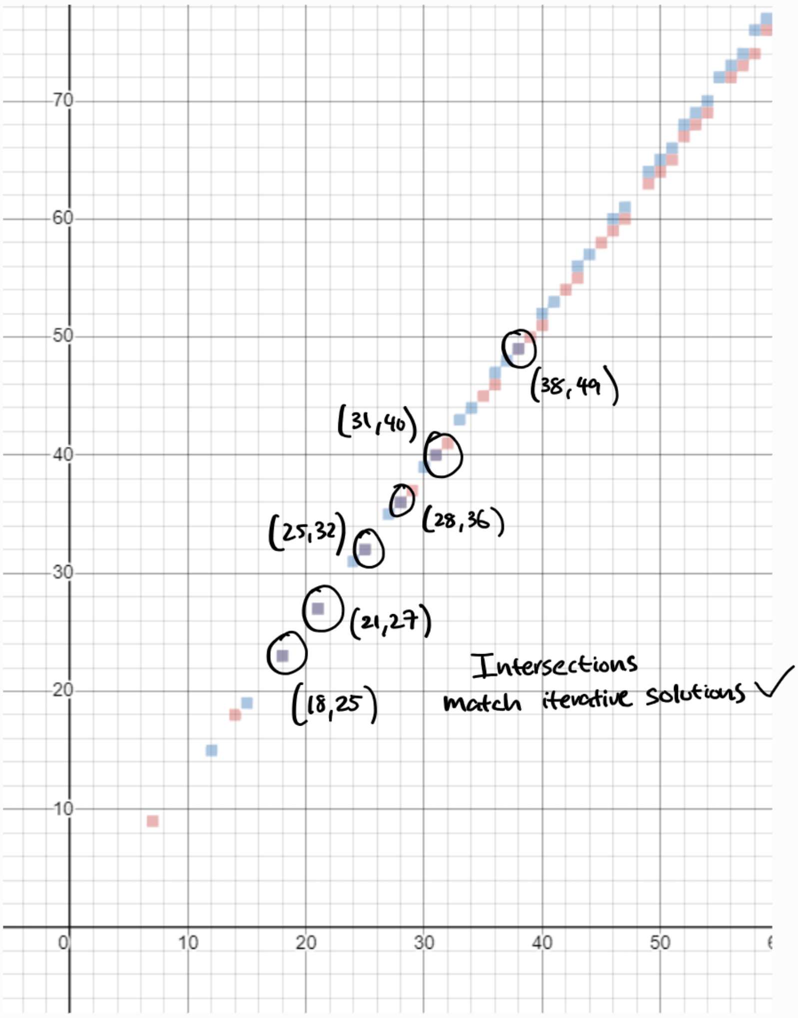
Previously, we defined the solutions for the problem to be:

$$\frac{775}{1000} \le \frac{s}{n} < \frac{785}{1000} \quad \text{AND} \quad \frac{755}{1000} \le \frac{s+1}{n+2} < \frac{765}{1000} \quad \text{WHERE } s \text{ and } n \text{ are whole numbers}$$

While you can plot double inequalities with a workaround, it's hard to know where points s and n are both whole numbers

You can use the floor, ceiling, or rounding to gaurantee whole numbers though.

With this in mind, you can use desmos to show you whole numbers which satisfy the inequality:



In this example:

$s = \text{round}(x)$

$n = \text{round}(y)$

$$\frac{775}{1000} \le \frac{s}{n} < \frac{785}{1000}$$

$$\frac{755}{1000} \le \frac{s+1}{n+2} < \frac{765}{1000}$$

Solutions (intersections)

The 4 inequalities intersect at the quadrilateral above, and diverge as s gets larger.