Take Home Quiz 4

AP Calculus BC

1. $f(x)=\frac{1}{x}$

$f'(x)=-\frac{1}{x^2}$

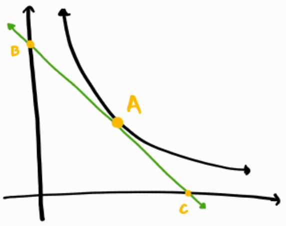
$y-y_1=m(x-x_1)$

$\therefore$ at $x=a$, the tangent line is:

$y-f(a)=f'(a)(x-a)$

$\boxed{y-\frac{1}{a}=-\frac{1}{a^2}(x-a)}$ ■

2.



a.

$y-\frac{1}{a}=-\frac{1}{a^2}(x-a)$

When $x=0$:

$y=-\frac{1}{a^2}(-a)+\frac{1}{a}$

$y=\frac{2}{a}$

$\boxed{B:\left(0,\frac{2}{a}\right)}$ ■

When $y=0$:

$-\frac{1}{a}=-\frac{1}{a^2}(x-a)$

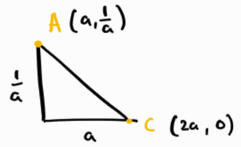
$-\frac{1}{a}=-\frac{x}{a^2}+\frac{1}{a}$

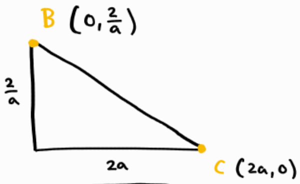
$-\frac{2}{a}=-\frac{x}{a^2}$

$x=2a$

$\boxed{C:(2a,0)}$ ■

b.





$\overline{AC}=\sqrt{a^2+\frac{1}{a^2}}$

$=\frac{1}{a}\sqrt{a^4+1}$

$\overline{BC}=\sqrt{4a^2+\frac{4}{a^2}}$

$=\frac{2}{a}\sqrt{a^4+1}$

$\overline{AC}:\overline{BC}$

$\frac{1}{a}\sqrt{a^4+1}:\frac{2}{a}\sqrt{a^4+1}$

$\boxed{1:2}$ ■

c. The ratio from 2b indicates that the midpoint of $\overline{BC}$ is at $x=a$, given $a\neq 0$.

5. a. $f(x)=\frac{1}{x^2}$

$f'(x)=-\frac{2}{x^3}$

$\boxed{y-\frac{1}{a^2}=-\frac{2}{a^3}(x-a)}$ ■

b. i. When $x=0$:

$y=\frac{2}{a^2}+\frac{1}{a^2}$

$=\frac{3}{a^2}$

$\boxed{B:\left(0,\frac{3}{a^2}\right)}$

When $y=0$:

$-\frac{1}{a^2}=-\frac{2}{a^3}(x-a)$

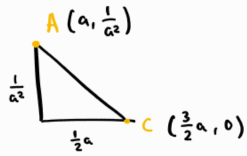
$-\frac{1}{a^2}=-\frac{2x}{a^3}+\frac{2}{a^2}$

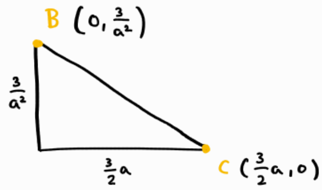
$-\frac{3}{a^2}=-\frac{2x}{a^3}$

$x=\frac{3}{2}a$

$\boxed{C:\left(\frac{3}{2}a,0\right)}$

ii.





$\overline{AC}=\sqrt{\frac{1}{a^4}+\frac{1}{4}a^2}$

$\overline{BC}=\sqrt{\frac{9}{a^4}+\frac{9}{4}a^2}$

$=3\sqrt{\frac{1}{a^4}+\frac{1}{4}a^2}$

$\overline{AC}:\overline{BC}$

$\sqrt{\frac{1}{a^4}+\frac{1}{4}a^2}:3\sqrt{\frac{1}{a^4}+\frac{1}{4}a^2}$

$\boxed{1:3}$ ■

iii. $\overline{BA}$ is now double $\overline{AC}$.

c. $A=\frac{1}{2}bh$

$=\frac{1}{2}\left(\frac{3}{2}a\right)\left(\frac{3}{a^2}\right)$

$\boxed{=\frac{9}{4a}}$ ■

The area of $\triangle BOC$ can be described as $A(x)=\frac{9}{4x}$, where x is the x-coordinate of such tangency

d. $A=lw$

$=(a)\left(\frac{1}{a^2}\right)$

$\boxed{=\frac{1}{a}}$ ■

Ratio of Rectangle to triangle: $\frac{1}{a}:\frac{9}{4a}=\boxed{4:9}$ ■

The area of the rectangle can be described as $A(x)=\frac{1}{x}$, where x is the x-coordinate of such tangency

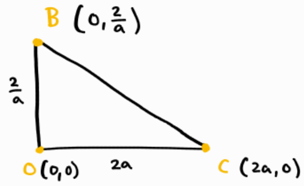
The ratio is not affected by a.

6. a. $f(x)=\frac{1}{x^n}$

$f'(x)=-\frac{n}{x^{n+1}}$

$\boxed{y-\frac{1}{a^n}=-\frac{n}{a^{n+1}}(x-a)}$ ■

3.



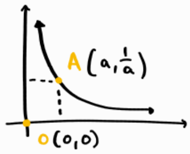
$A = \frac{1}{2}bh$

$= \frac{1}{2}(2a)(\frac{2}{a})$

$\boxed{=2}$ ■

The area of ΔBOC is always 2, regardless of where the point of tangeny $(a, \frac{1}{a})$ is (except $a=0$)

4.



$A = lw$

$= a \cdot \frac{1}{a}$

$\boxed{=1}$ ■

Ratio of Rectangle to triangle: $\boxed{1:2}$

The area of the rectangle is always 1, regardless of where the point of tangency $(a, \frac{1}{a})$ is (except $a=0$)

The ratio is not affected by a.

b. i. When $x=0$:

$y = \frac{an}{a^{n+1}} + \frac{1}{a^n}$

$= \frac{n}{a^n} + \frac{1}{a^n}$

$= \frac{n+1}{a^n}$

$\boxed{B: \left(0, \frac{n+1}{a^n}\right)}$ ■

When $y=0$

$-\frac{1}{a^n} = -\frac{nx}{a^{n+1}} + \frac{n}{a^n}$

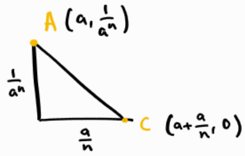
$\frac{n+1}{a^n} = \frac{nx}{a^{n+1}}$

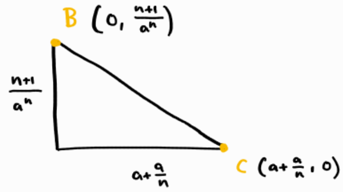
$nx = a(n+1)$

$x = a + \frac{a}{n}$

$\boxed{C: \left(a + \frac{a}{n}, 0\right)}$ ■

ii.





$\overline{AC} = \sqrt{\frac{1}{a^{2n}} + \frac{a^2}{n^2}}$

$\overline{BC} = \sqrt{\frac{(n+1)^2}{a^{2n}} + \frac{a^2(n+1)^2}{n^2}}$

$= (n+1)\sqrt{\frac{1}{a^{2n}} + \frac{a^2}{n^2}}$

$a + \frac{a}{n}$

$= \frac{an+a}{n}$

$= \frac{a(n+1)}{n}$

$\overline{AC} : \overline{BC}$

$\sqrt{\frac{1}{a^{2n}} + \frac{a^2}{n^2}} : (n+1)\sqrt{\frac{1}{a^{2n}} + \frac{a^2}{n^2}}$

$\boxed{1 : n+1}$ ■

iii. $\overline{BC}$ is $n+1$ times larger than $\overline{AC}$

c. $A = \frac{1}{2}bh$

$= \frac{1}{2}\left(\frac{a(n+1)}{n}\right)\left(\frac{n+1}{a^n}\right)$

$\boxed{= \frac{(n+1)^2}{2na^{n-1}}}$

The area of ΔBOC can be described as $A(x) = \frac{(n+1)^2}{2na^{n-1}}$, where x is the x-coordinate of such tangency

d. $A = lw$

$= a\left(\frac{1}{a^n}\right)$

$\boxed{= a^{1-n}}$ ■

Ratio of Rectangle to triangle: $a^{1-n} : \frac{(n+1)^2}{2na^{n-1}} = \boxed{2n : (n+1)^2}$ ■

The area of the rectangle can be described as $A(x) = a^{1-n}$, where x is the x-coordinate of such tangency

The ratio is not affected by a.