Chapter 8

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I. Angular Quantities
   a. Axis of rotation: The straight line through all fixed points of a rotating rigid body around which all other points of the body move in circles
   b. Radian: The angle subtend by an arc whose length is equal to the radius
      i. $\theta = \frac{l}{r}$
      ii. $360° = 2\pi\ rad$
   c. Angular Velocity
      i. Average Angular Velocity: $\omega(= \frac{\Delta!}{\Delta"}$
      ii. Instantaneous Angular Velocity: $\omega = \lim_{\Delta" \to 0} \frac{\Delta!}{\Delta"}$
   d. Angular Acceleration
      i. Average Angular Acceleration: $\alpha* = \frac{\Delta\#}{\Delta"}$
      ii. Instantaneous Angular Acceleration: $\alpha = \lim_{\Delta" \to 0} \frac{\Delta\#}{\Delta"}$
   e. Linear Quantities:
      i. Velocity: $v = r\omega$
      ii. Rotational Acceleration: $a_{\$} = \frac{\%^2}{\&} = \omega^2 r$
      iii. Tangential Acceleration: $a_{'''(} = r\alpha$

II. Constant Angular Acceleration
   i. Angular and Linear Equation Relationships
   Note: θ is in radians.

Angular	**Linear**
$\omega = \omega_{)} + \alpha t$	$v = v_0 + at$
$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x = x_0 + v_0 t + \frac{1}{2}at^2$
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$
$\omega(= \frac{\omega + \omega_0}{2}$	$\bar{v} = \frac{v + v_0}{2}$

III. Rolling Motion (Without Slipping)
   a. Rolling without slipping: $v = r\omega$

IV. Torque
   a. Lever arm: The perpendicular distance from the axis of rotation to the line along which the force acts
   b. Torque: $\tau = rF_* = rF\sin\theta$

V. Rotational Dynamics; Torque and Rotational Inertia
   a. Torque of single particle: $\tau = mr^2\alpha$
   b. Net torque: $\Sigma\tau = I\alpha$

VI. Solving Problems in Rotational Dynamics

VII. Rotational Kinetic Energy
   a. Kinetic Energy of rotating object: $KE = \frac{\pm}{,} I\omega'$
   b. Kinetic Energy of system (linear + rotational): $\Sigma KE = \frac{\pm}{,} mv' + \frac{\pm}{,} I\omega'$
   c. Work done by Torque: $W = \tau\Delta\theta$

VIII. Angular Momentum and Its Conservation
   a. Law of conservation of momentum: The total angular momentum of a rotating object remains constant if the net torque acting on it is zero

IX. Vector Nature of Angular Quantities
   a. Right hand rule: When the fingers of the right hand are curled around the rotation axis and point in the direction of the rotation, then the thumb points in the direction $\omega 99^{\rightarrow}$
   b. Angular Momentum: $\mathcal{L}^{\rightarrow} = I\omega 99^{\rightarrow} = mvr = \tau\Delta t$

Relevant Equations:

- Arc Length: $\pi d \times \frac{!}{-./}$

Units:

- ω: $\frac{rads}{s}$
- α: $\frac{rads}{s'}$
- τ: $m\ A\ N$
- I: $kg\ A\ m$

Momentum of Inertia Equations

Object	**Location of Axis**	**Variables**	**Equation**
Thin hoop	Through center	Radius: r	mr^2
Thin hoop	Through central diameter	Radius: r Width: w	1/2 mr^2+1/12 mw^2
Solid Cylinder	Through center	Radius: r	1/2 mr^2
Hollow Cylinder	Through center	Inner Radius: r+ Outer Radius: r,	1/2 m(r_1^2+r_2^2)
Uniform Sphere	Through center	Radius: r	2/5 mr^2
Long Uniform Rod	Through center	Length: l	1/12 ml^2
Long Uniform Rod	Through end	Length: l	1/3 ml^2
Rectangular Thin Plate	Through center	Length: l Width: w	1/12 m(l^2+w^2)
Satellite	Through end	Radius: r	mr'

Linear and Rotational Relationships

Linear	**Type**	**Rotational**	**Relation**
x	Displacement	θ	$x = r\theta$
v	Velocity	ω	$v = r\omega$
$a_{'''(}$	Acceleration	α	$a_{'''(} = r\alpha$