7 - Rational Functions

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I. Inverse Variation
   a. Classifying Equations:

None	x
Direct	ax
Inverse	$\frac{a}{x}; x \neq 0$

   b. Classifying Data:

None	xy and $\frac{y}{x}$ are not constant
Direct	$\frac{y}{x}$ is constant
Inverse	xy is constant

II. Graphing Rational Functions
   a. Rational Function Form: $f(x) = \frac{p(x)}{q(x)}$
      i. Simple Rational Function: $y = \frac{a}{x}$
         1) Has the same asymptotes, domain and range as $f(x) = \frac{1}{x}$
         2) Graphing Translations with $y = \frac{a}{x-h} + k$

Step 1	Draw the asymptotes $x = h$ and $y = k$
Step 2	Plot points to the left and to the right of the vertical asymptote
Step 3	Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes

      ii. Other Rational Functions: $y = \frac{ax+b}{cx+d}$
         1) Vertical Asymptote: $x = -\frac{d}{c}$
         2) Horizontal Asymptote: $y = \frac{a}{c}$
         3) Covert $y = \frac{ax+b}{cx+d}$ to $y = \frac{a}{x-h} + k$ using long division or sythentic division

III. Multiplying and Dividing Rational Expressions
   a. Rational Expression: A fraction whose numerator and denominator are nonzero polynomials
   b. Simplified Form: When a ration expression's numerator and denominator have no common factors
   c. Simplifying Rational Expressions: $\frac{ac}{bc} = \frac{a}{b}$
      i. Tip: Factor the polynomial to find common factors
   d. Multiplying Rational Expressions: $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}; b, d \neq 0$
      i. Tip: Factor the polynomial to find common factors to simplify
   e. Dividing Rational Expressions: $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}; b, c, d \neq 0$
      i. Tip: Factor the polynomial to find common factors to simplify

IV. Adding and Subtracting Rational Expressions
   a. Adding and Subtracting with Like Denominators:
      i. Addition: $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}; c \neq 0; d \neq 0$
      ii. Subtraction: $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$

b. Adding and Subtracting with Unlike Denominators:
    i. Addition: $\frac{a}{c}+\frac{b}{d}=\frac{ad+bc}{cd}$
    ii. Subtraction: $\frac{a}{c}-\frac{b}{d}=\frac{ad-bc}{cd}$

V. Solving Rational Equations
    a. Cross multiplication: $\frac{a}{b}=\frac{c}{d}; ad=bc$

8 - Sequences and Series

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I. Defining and Using Sequences and Series
   a. Sequence: An ordered list of numbers
      i. Domain: Relative position of each term
      ii. Range: Terms of the sequence
   b. Series: When the terms of a sequence are added together
      i. Summation Notation (Sigma Notation): Used to write a series
         1) $\sum_{b=c}^{a} d$

Symbol	Name
a	Upper limit of Summation
b	Index of Summation
c	Lower limit of summation
d	Rule

      ii. Formulas for Special Series

Sum of n terms of 1	$\sum_{i=1}^{n} 1 = n$
Sum of first n positive integers	$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
Sum of squares of first n positive integers	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

II. Analyzing Arithmetic Sequences and Series
   a. Arithmetic Sequence: Where the difference between consecutive terms is constant (common difference or d)
      i. Testing if a sequence is arithmetic: $d = a_{n+1} - a_n d$ (d should be constant)
      ii. Rule for Arithmetic Sequences: $a_n = a_1 + (n-1)d$

Symbol	Name
a_n	The nth term
a_1	The first term
d	The common difference

   b. Arithmetic Series: The expression formed by adding the terms of an arithmetic sequence
      i. The Sum of a Finite Arithmetic Series: $S_n = n(\frac{a_1+a_n}{2})$

III. Analyzing Geometric Sequences and Series
   a. Geometric Sequence: Where the ratio of any term to the previous term is constant (common ratio or r)
      i. Testing if a sequence is geometric: $r = \frac{a_{n+1}}{a_n}$ (r should be constant)
      ii. Rule for Geometric Sequences: $a_n = a_1 r^{n-1}$

Symbol	Name

a_n	The nth term
a_1	The first term
r	The common ratio

b. Geometric Sequence: The expression formed by adding the terms of a geometric sequence
    i. The Sum of a Finite Geometric Sequence: $S_n = a_1\left(\frac{1-r^n}{1-r}\right); r \neq 1$

IV. Finding Sums of Infinite Geometric Series
a. Partial Sum: The sum S_n to the first n terms of an infinite series (may be approaching a limiting value)
b. The Sum of an Infinite Geometric Series: $S = \frac{a_1}{1-r}; |r| < 1$

V. Using Recursive Rules with Sequences
a. Explicit Rule: Gives a_n as a function of the term's position n in the sequence
b. Recursive Rule: Gives the beginning term(s) of a sequence
c. Recursive Equation: Tells how a_n is related to one or more preceding terms
    i. Recursive Equations for Arithmetic and Geometric Sequences
        1) Arithmetic Sequence: $a_n = a_{n-1} + d$
        2) Geometric Sequence: $a_n = ra_n - 1$

9 - Trigonometric Ratios and Functions

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I. Right Triangle Trigonometry
   a. Right Triangle Definitions of Trigonometric Functions

$\sin\theta = \frac{Opposite}{Hypotenuse}$	$\cos\theta = \frac{Adjacent}{Hypotenuse}$	$\tan\theta = \frac{Opposite}{Adjacent}$
$\csc\theta = \frac{Hypotenuse}{Opposite} = \frac{1}{\sin\theta}$	$\sec\theta = \frac{Hypotenuse}{Adjacent} = \frac{1}{\cos\theta}$	$\cot\theta = \frac{Adjacent}{Opposite} = \frac{1}{\tan\theta}$

II. Angles and Radian Measure
   a. Standard Position: When an angle's vertex is at the origin and its initial side lies on the positive x-axis
   b. Coterminal: When an angles terminal sides coincide
   c. Radian: The measure of an angle in standard position whose terminal side intercepts an arc of length r
   d. Converting Between Degrees and Radians
      i. Degrees to Radians: $\frac{\pi}{180}$
      ii. Radians to Degrees: $\frac{180°}{\pi}$
   e. Sector: A region of a circle that is bounded by two radii and an arc of the circle
   f. Central Angle: The angle θ formed by the sector
   g. Arc Length and Area of Sector:
      i. Arc Length: $s = r\theta$
      ii. Area: $A = \frac{1}{2}r^2\theta$

III. Trigonometric Functions of Any Angle
   a. General Definitions of Trigonometric Functions

$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{y}{r}; y \neq 0$
$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}; x \neq 0$
$\tan\theta = \frac{y}{x}; x \neq 0$	$\cot\theta = \frac{x}{y}; y \neq 0$

   b. Unit Circle: The circle $x^2 + y^2 = 1$ which has center $(0,0)$ and a radius 1
   c. Quadrantal Angles: An angle in standard position whose terminal side lies on an axis
   d. Reference Angle Relationships

	Quadrant 2 (Top Left)	**Quadrant 3 (Bottom Left)**	**Quadrant 4 (Bottom Right)**
Degrees	$180 - \theta$	$\theta - 180$	$360 - \theta$
Radians	$\pi - \theta$	$\theta - \pi$	$2\pi - \theta$

   e. Evaluating Trigonometric Functions

Quadrant II $\sin\theta, \csc\theta$ $-\cos\theta, -\sec\theta$ $-\tan\theta, -\cot\theta$	**Quadrant I** $\sin\theta, \csc\theta$ $\cos\theta, \sec\theta$ $\tan\theta, \cot\theta$
Quadrant III $-\sin\theta, -\csc\theta$ $-\cos\theta, -\sec\theta$ $\tan\theta, \cot\theta$	**Quadrant IV** $-\sin\theta, -\csc\theta$ $\cos\theta, \sec\theta$ $-\tan\theta, -\cot\theta$

IV. Graphing Sine and Cosine Functions
   a. Characteristics of $y = \sin(x)$ and $y = \cos(x)$
      i. Domain: All real numbers
      ii. Range: $-1 \leq y \leq 1$
      iii. Amplitude: 1
      iv. Period: 2π
   b. Amplitude and Period of $y = a\sin(bx)$ and $y = a\cos(bx)$
      i. Amplitude: $|a|$
      ii. Period: $\frac{2\pi}{|b|}$
   c. Phase Shift: A horizontal translation of a periodic function (Note: You may need to factor out b to view h)
   d. Graphing $y = a\sin b(x-h) + k$ and $y = a\cos b(x-h) + k$

Step 1	Identify the amplitude a, the period $\frac{2\pi}{b}$, the horizontal shift h, and the vertical shift k of the graph
Step 2	Draw the horizontal line $y = k$
Step 3	Find the five key points by translating the key points of $y = a\sin(bx)$ or $y = a\cos(bx)$ horizontally h units and vertically k units
Step 4	Draw the graph through five translated key points

V. Graphing Other Trigonometric Functions
   a. Characteristics of $y = \tan(x)$ and $y = \cot(x)$
      i. Domain
         1) $\tan(x)$: All real numbers except odd multiples of $\frac{\pi}{2}$
         2) $\cot(x)$: All real numbers except multiples of π
      ii. Range: All real numbers
      iii. Period: π
   b. Period and Vertical Asymptotes of $y = a\tan(bx)$ and $y = a\cot(bx)$
      i. Period: $\frac{\pi}{b}$
      ii. Vertical Asymptote
         1) $y = a\tan(bx)$: Odd multiples of $\frac{\pi}{|b|}$
         2) $y = a\cot(bx)$: Multiples of $\frac{\pi}{|b|}$
   c. Characteristics of $y = \sec(x)$ and $y = \csc(x)$
      i. Domain
         1) $y = \sec(x)$: All real numbers except odd multiples of $\frac{\pi}{2}$
         2) $y = \csc(x)$: All real numbers except multiples of π
      ii. Range: $-1 \geq y \geq 1$
      iii. Period: 2π

VI. Modeling with Trigonometric Functions
   a. Frequency: The number of cycles per unit of time, which is the reciprocal of the period
   b. Sinusoids: Graphs of sine and cosine functions
      i. Writing sine and cosine functions: Find all values of a, b, h, and k

VII. Using Trigonometric Identities
   a. Trigonometric Identity: A trigonometric equation that is true for all values of the variable for which both sides of the equation are defined
   b. Fundamental Trigonometric Identities

Reciprocal Identities	$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$
Tangent and Cotangent Identities	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	

Pythagorean Identities	$\sin(\theta)^2 + \cos(\theta)^2 = 1$	$1 + \tan(\theta)^2 = \sec(\theta)^2$	$1 + \cot(\theta)^2 = \csc(\theta)^2$
Cofunction Identities	$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$	$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$	$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$
Negative Angle Identities	$\sin(-\theta) = -\sin\theta$	$\cos(-\theta) = \cos\theta$	$\tan(-\theta) = -\tan\theta$

VIII. Using Sum and Difference Formulas

a. Sum and Difference Formulas

Sum Formulas	**Difference Formulas**
$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$	$\sin(a - b) = \sin(a)\cos(b) - \cos(a)\sin(b)$
$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$	$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)$
$\tan(a + b) = \frac{\tan(a) + \tan(b)}{1 - \tan(a)\tan(b)}$	$\tan(a - b) = \frac{\tan(a) - \tan(b)}{1 + \tan(a)\tan(b)}$

10 - Probability

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I. Sample Spaces and Probability
   a. Probability Experiment: An action, or trial, that has varying results
   b. Outcomes: The possible result of a probability experiment
   c. Event: A collection of one or more outcomes in a probability experiment
   d. Sample Space: The set of all possible outcomes for an experiment
   e. Probability of Event $P(A)$: A measure of the likelihood, or chance that an event will occur
   f. Theoretical Probability: $P(A) = \frac{O_f}{O_t}$

Symbol	Name
O_f	Number of favorable outcomes
O_t	Total number of outcomes

   g. Probability of the Complement of an Event: $P(\overline{A}) = 1 - P(A)$
   h. Geometric Probability: A probability found by calculating a ratio of two lengths, areas, or volumes
   i. Experimental Probability: $P(A) = \frac{N_s}{N_t}$

Symbol	Name
N_s	Number of successes
N_t	Number of trials

II. Independent and Dependent Events
   a. Independent Events: Two events in which the occurrence of one event does not affect the occurrence of another event
      i. Probability of Independent Events: $P(A\&B) = P(A) \times P(B)$
   b. Dependent Events: Two events in which the occurrence of one event does affect the occurrence of the other event
      i. Conditional Probability $P(B|A)$: The probability that $P(B)$ occurs given that $P(A)$ has occurred
      ii. Probability of Dependent Events: $P(A\&B) = P(A) \times P(B|A)$

III. Two-Way Tables and Probability
   a. Two-Way Table: A frequency table that displays data collected from one source that belongs to two different categories
      i. Joint Frequency: Each entry in a two-way table
      ii. Marginal Frequencies: The sum of the rows and columns in a two-way table
   b. Relative and Conditional Relative Frequencies
      i. Joint Relative Frequency: The ratio of frequency that is not in the total row or the total column to the total number of values or observations in a two-way table
      ii. Marginal Relative Frequency: The sum of the join relative frequency in a row or a column in a two-way table
      iii. Conditional Relative Frequency: A ratio of a joint relative frequency to the marginal relative frequency in a two-way table

IV. Probability of Disjoint and Overlapping Events
   a. Compound Event: The union or intersection of two events
      i. Disjoint/Mutually Exclusive: Two events that have no outcomes in common
      ii. Probability of Compound Events:
         1) Any two events: $P(A||B) = P(A) + P(B) - P(A\&B)$
         2) Disjoint/Mutually Exclusive Events: $P(A||B) = P(A) + P(B)$

V. Permutations and Combinations
   a. n Factorial: The product of the integer range 1 to n, for any positive integer n
   b. Permutations: An arrangement of objects in which order is important
      i. The number of permutations of n objects: $nP_n = n!$
      ii. The number of permutations of n objects taken r at a time: $nP_r = \frac{n!}{(n-r)!}$
   c. Combinations: A selection of objects in which order does not important
      i. The number of combinations of n objects taken r at a time: $nC_r = \frac{n!}{(n-r)!\times r!}$
   d. Binomial Theorem: $(a+b)^n = nC_0 a^n b^0 + \cdots + nC_r a^{n-r} b^r$

VI. Binomial Distributions
   a. Random Variable: A variable whose value is determined by the outcomes of a probability experiment
   b. Probability Distribution: A function that gives the probability of each possible value of a random variable
   c. Binomial Distribution: A type of probability distribution that shows the probabilities of the outcomes of a binomial experiment
   d. Binomial Experiment: An experiment in which there are a fixed number of independent trials, exactly two possible outcomes for each trial, and the probability of success is the same for each trial
      i. The probabiltiy of exactly k successes in n trials: $P(k) = nC_k p^k (1-p)^{n-k}$

11 - Data Analysis and Statistics

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I. Using Normal Distributions
   a. Normal Distribution: A bell-shaped curve called a normal curve that is symmetric about the mean
   b. Areas Under a Normal Curve

Areas of $\mu \pm z\alpha$	**Percentage**
Total	100%
$x = 1$	68%
$x = 2$	95%
$x = 3$	99.7%
Areas of $\mu \pm z\alpha$	**Percentage**
$x = 1$	34%
$x = 2$	13.5%
$x = 3$	2.35%
$x = 4$	0.15%

   c. Normal Distributions:
      i. Bell Shaped and Symmetric
         1) Histogram has a normal distributions
         2) Mean = Median
      ii. Skewed Left
         1) Histogram does not have normal distribution
         2) Mean < Median (Most data is to the right of the mean)
      iii. Skewed Right
         1) Histogram does not have normal distribution
         2) Mean > Median (Most data is to the left of the mean)

II. Populations, Samples, and Hypotheses
   a. Population: The collection of all data, such as response, measurements, or counts, that you want information about
   b. Sample: A subset of a population
   c. Sample Space: All possible combinations
   d. Parameter: A numerical description of a population characteristic (Typically denoted with "all")
   e. Statistic: A numerical description of a sample characteristic (Typically denoted with a "some")

III. Collecting Data
   a. Random Sample: A sample in which each member of a population has an equal chance of being selected
   b. Types of Samples

Sample Name	**Description**
Self-Selected Sample	A sample in which members of a population can volunteer to be in the sample
Systematic Sample	A sample in which a rule is used to select members of a population

Stratified Sample	A sample in which a population is divided into smaller groups that share a similar characteristic and a sample is then randomly selected from each group
Cluster Sample	A sample in which a population is divided into groups, called clusters, and all of the members in one or more of the clusters are randomly selected
Convenience Sample	A sample in which only members of a population who are easy to reach are selected

c. Recognizing Bias in Sampling
   i. Bias: An error that results in a misrepresentation of a population
   ii. Unbiased Sample: A sample that is representative of the population that you want information about
   iii. Biased Sample: A sample that overrepresents or underrepresents part of the population

d. Methods of Collecting Data

Method Name	**Description**
Experiment	A method that imposes a treatment on individuals in order to collect data on their response to the treatment
Observational Study	Individuals are observed and variables are measured without controlling the individuals or their environment
Survey	An investigation of one or more characteristics of a population
Simulation	The use of a model to reproduce the conditions of a situation or process so that the simulated outcomes closely match the real-world outcomes

e. Recognizing Bias in Survey Questions
   i. Biased Question: A question that is flawed in a way that leads to inaccurate results
      1) Encourage a particular response
      2) Are too sensitive to answer truthfully
      3) Do not provide enough information to give an accurate opinion
      4) Address more than one issue

IV. Experimental Design
   a. Controlled Experiment: An experiment in which two groups are studied under identical conditions with the exception of one variable
      i. Control Group: The group under ordinary conditions that is subjected to no treatment during an experiment
      ii. Treatment Group: The group that is subjected to the treatment in an experiment
   b. Randomized Comparative Experiment: An experiment in which subjects are randomly assigned to the control group or the treatment group
      i. Randomization: A process of randomly assigning subjects to different treatment groups
      ii. Placebo: A harmless, unmedicated treatment that resembles the actual treatment
   c. Randomized Comparative Experiment vs. Observational Studies
      i. Randomized Comparative Experiments can make valid cause-and-effect conclusions
      ii. Observational Studies can identify correlation between variables, but not causality
   d. Replication: The repetition of an experiment under the same or similar conditions to improve the validity of the experiment

V. Making Inferences from Sample Surveys
   a. Descriptive Statistics: The branch of statistics that involves the organization, summarization, and display

b. Inferential Statistics: The branch of statistics that involves using a sample to draw conclusions about a population
c. Margin of Error: The limit on how much the responses of the sample would differ from the responses of the population
    i. Margin of Error for a random sample of size n: $\pm\frac{1}{\sqrt{n}}$

VI. Making Inferences from Experiments
a. Resampling data using a simulation

Step 1	Assign each value a integer key
Step 2	Randomly sort the keys
Step 3	Using the same index, retrieve the new sorted keys

Conics

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I. Hyperbolas
   a. Hyperbola: The set of all points P in a plane such that the difference of the distances between P and two fixed points (foci) are constant
      i. Vertex: The point where the line through the foci intersects the hyperbola
      ii. Transverse Axis: The axis that joins the vertices
      iii. Standard Equation of a Hyperbola with Center at the Origin

Equation	Transverse Axis	Asymptotes	Vertices	Focus
$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	Horizontal	$y=\pm\frac{b}{a}x$	$(\pm a,0)$	$(\pm\sqrt{a^2+b^2},0)$
$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$	Vertical	$y=\pm\frac{a}{b}x$	$(0,\pm a)$	$(0,\pm\sqrt{a^2+b^2})$

         1) Quickly solving for Transverse Axis using term comparison
            a) Horizontal: $y_t < 0$
            b) Vertical: $y_t > 0$

II. Ellipses
   a. Ellipse: The set of all points P in a plane such that the sum of the distances between P and two fixed points (foci) are constant
      i. Vertex: The point where the line through the foci intersects the ellipse
      ii. Major Axis: The axis that joins the vertices
      iii. Co-Vertices: The point where the line perpendicular to the major axis intersects the ellipse
      iv. Minor Axis: The axis that joins the co-vertices
      v. Standard Equation of an Ellipse with Center at the Origin

Equation	Major Axis	Vertices	Co-Vertices	Focus
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	Horizontal	$(\pm a,0)$	$(0,\pm b)$	$(\pm\sqrt{a^2-b^2},0)$
$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	Vertical	$(0,\pm a)$	$(\pm b,0)$	$(0,\pm\sqrt{a^2-b^2})$

         1) Quickly solving for Major Axis using denominator comparison
            a) Horizontal: $x_d > y_d$
            b) Vertical: $y_d > x_d$

III. Conic Sections
   a. Conic Sections/Conics: Formed when a plane intersects a double-napped cone
   b. Standard Form of Equations of Translated Conics

Circle	$(x-h)^2+(y-k)^2=r^2$

	Horizontal Axis	Vertical Axis
Parabola	$(y-k)^2=4p(x-h)$	$(x-h)^2=4p(y-k)$
Ellipse	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
Hyperbola	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$

   c. Classifying Conics Using Their Equations
      i. General Second-Degree Equation: $ax^2+bxy+cy^2+dx+ey+f=0$

Discriminant	**Type of Conic**
$b^2 - 4ac < 0; b = 0; a = c$	Circle
$b^2 - 4ac < 0; b \neq 0 \| \| a \neq c$	Ellipse
$b^2 - 4ac = 0$	Parabola
$b^2 - 4ac > 0$	Hyperbola

Misc.

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I. Long Division (Polynomials)

Step 1	Divide the leading coefficient of the dividend with the leading coefficient of the divisor and add it to the quotient
Step 2	Multiply the result from **Step 1** with the divisor and add it to the remainder
Step 3	Subtract the remainder
Step 4	Repeat except using the remainder from **Step 3** until not divisible

II. Synthetic Division (Polynomials)

Step 1	Put coefficients of the dividend each in the box in descending order (i.e. $ax^4 + bx^3 + cx^2$ …)
Step 2	Solve for x in the divisor and put the answer outside the box
Step 3	Bring down the first coefficient, and multiply the coefficient with the solution from **Step 2**
Step 4	Add the answer from **Step 3** to the following coefficient, and repeat **Step 3** except by using the following coefficient

III. Completing the Square: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$