

# Chapter 1: Linear Functions

Wednesday, May 12, 2021 9:36 AM

- I. 1.1: Parent Functions and Transformations
  - a. Transformation: Changes the size, shape, position, or orientation of a graph
  - b. Translation: A transformation that shifts a graph horizontally and/or vertically but does not change its size, shape, or orientation
  - c. Reflection: A transformation that flips a graph over a line called the line of reflection
  - d. Vertical Stretch: When the factor to multiply the y-coordinate is greater than 1
  - e. Vertical Shrink: When the factor to multiply the y-coordinate is greater than 0 and less than 1
- II. 1.2: Transformations of Linear and Absolute Value Functions
  - a. Horizontal Translations by  $h$ :  $g(x) = f(x - h)$
  - b. Vertical Translations by  $k$ :  $g(x) = f(x) + k$
  - c. Reflection in the x-axis:  $g(x) = -f(x)$
  - d. Reflection in the y-axis:  $g(x) = f(-x)$
  - e. Horizontal Stretches by  $\frac{1}{a}$ :  $g(x) = f(ax)$

Shrink	$0 < a < 1$
Stretch	$a > 1$

# Chapter 8: Sequences and Series

Monday, February 1, 2021 5:51 PM

## I. Lesson 1: Defining and Using Sequences

- a. Sequence: Ordered list of numbers
- b. Formulas for Special Series

Sum of $n$ terms of 1	$\sum_{i=1}^n 1 = n$
Sum of first $n$ positive integers	$\sum_{i=1}^n i = \frac{n(n+1)}{2}$
Sum of squares of first positive integers	$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$

## II. Lesson 2: Analyzing Arithmetic Sequences and Series

- a. Arithmetic Sequence: The difference of consecutive terms is constant
- b. Rules for Arithmetic Sequences

Equation	$a_n = a_1 + (n-1)d$	$d$ is the common difference
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- c. The Sum of a Finite Arithmetic Series

Equation	$S_n = n\left(\frac{a_1 + a_n}{2}\right)$
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## III. Lesson 3: Analyzing Geometric Sequences and Series

- a. Geometric Sequence: The ratio of any term to the previous term is constant
- b. Rules for Geometric Sequences

Equation	$a_n = a_1 r^{n-1}$	$r$ is the common ratio
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- c. The Sum of a Finite Geometric Series

Equation	$S_n = a_1 \left(\frac{1-r^n}{1-r}\right)$	$r$ is the common ratio
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## IV. Lesson 4: Finding Sums of Infinite Geometric Series

- a. The Sum of an Infinite Geometric Series

Equation	$S = \frac{a_1}{1-r}$
Restrictions	$ r  < 1$

## V. Lesson 5: Using Recursive Rules with Sequences

- a. Recursive Equations for Arithmetic and Geometric Sequences

<b>Arithmetic Sequence</b>	$a_n = a_{n-1} + d$	$d$ is the common difference
<b>Geometric Sequence</b>	$a_n = r a_{n-1}$	$r$ is the common ratio

# Chapter 9: Trigonometric Ratios and Functions

Monday, April 19, 2021 9:43 PM

## I. Right Triangle Trigonometry

### a. Right Triangle Definitions of Trigonometric Functions

Name	Equation	Relationships
Sine	$\sin \theta = \frac{\textit{opposite}}{\textit{hypotnuse}}$	$\sin \theta = \frac{1}{\textit{csc } \theta}$
Cosine	$\cos \theta = \frac{\textit{adjacent}}{\textit{hypotnuse}}$	$\cos \theta = \frac{1}{\textit{sec } \theta}$
Tangent	$\tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$	$\tan \theta = \frac{1}{\textit{cot } \theta}$
Cosecant	$\textit{csc } \theta = \frac{\textit{hypotnuse}}{\textit{opposite}}$	$\textit{csc } \theta = \frac{1}{\sin \theta}$
Secant	$\textit{sec } \theta = \frac{\textit{hypotnuse}}{\textit{adjacent}}$	$\textit{sec } \theta = \frac{1}{\cos \theta}$
Cotangent	$\textit{cot } \theta = \frac{\textit{adjacent}}{\textit{opposite}}$	$\textit{cot } \theta = \frac{1}{\tan \theta}$

### b. Trigonometric Values for Special Angles

$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\textit{csc } \theta$	$\textit{sec } \theta$	$\textit{cot } \theta$
$30^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

## II. Angles and Radian Measure

- Standard Position: When vertex is at the position and the initial side lies on the positive x-axis
- Converting between degrees and radians

Degree to Radians	Radians to Degrees
$\theta \times \frac{\pi}{180}$	$\theta \times \frac{180}{\pi}$

- Arc Length:  $s = r\theta$  (Radians)
- Area:  $A = \frac{1}{2}r^2\theta$  (Radians)

## III. Trigonometric Function of Any Angle

- General definitions of trigonometric functions

$$r = \sqrt{x^2 + y^2}$$

$\sin \theta = \frac{y}{r}$	$\textit{csc } \theta = \frac{r}{y}; y \neq 0$
$\cos \theta = \frac{x}{r}$	$\textit{sec } \theta = \frac{r}{x}; x \neq 0$
$\tan \theta = \frac{y}{x}; x \neq 0$	$\textit{cot } \theta = \frac{x}{y}; y \neq 0$

b. Unit Circle: The circle  $x^2 + y^2 = 1$  which has the center  $(0,0)$ , and radius 1

c. Reference Angle Relationships

	Quadrant 2 (Top Left)	Quadrant 3 (Bottom Left)	Quadrant 4 (Bottom Right)
Degrees	$180 - \theta$	$\theta - 180$	$360 - \theta$
Radians	$\pi - \theta$	$\theta - \pi$	$2\pi - \theta$

IV. Lesson 7: Using Trigonometry Identities

a. Fundamental Trigonometric Identities

Reciprocal Identities	$\csc \theta = \frac{1}{\sin \theta}$	$\sec \theta = \frac{1}{\cos \theta}$	$\cot \theta = \frac{1}{\tan \theta}$
Tangent and Cotangent Identities	$\tan \theta = \frac{\sin \theta}{\cos \theta}$	$\cot \theta = \frac{\cos \theta}{\sin \theta}$	
Pythagorean Identities	$\sin(\theta)^2 + \cos(\theta)^2 = 1$	$1 + \tan(\theta)^2 = \sec(\theta)^2$	$1 + \cot(\theta)^2 = \csc(\theta)^2$
Cofunction Identities	$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$	$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$	$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$
Negative Angle Identities	$\sin(-\theta) = -\sin \theta$	$\cos(-\theta) = \cos \theta$	$\tan(-\theta) = -\tan \theta$

b. Determining Positivity of Trigonometric Functions with Quadrants

<b>Quadrant II</b> $\sin \theta, \csc \theta$ $-\cos \theta, -\sec \theta$ $-\tan \theta, -\cot \theta$	<b>Quadrant I</b> $\sin \theta, \csc \theta$ $\cos \theta, \sec \theta$ $\tan \theta, \cot \theta$
<b>Quadrant III</b> $-\sin \theta, -\csc \theta$ $-\cos \theta, -\sec \theta$ $\tan \theta, \cot \theta$	<b>Quadrant IV</b> $-\sin \theta, -\csc \theta$ $\cos \theta, \sec \theta$ $-\tan \theta, -\cot \theta$

V. Lesson 8: Using Sum and Difference Formulas

# Chapter 10: Probability

Tuesday, February 16, 2021 5:28 PM

## I. Lesson 1: Sample Spaces and Probability

- Probability Experiment: An action or trial that has varying results
- Outcomes: Possible results of a probability experiment
- Event: Collection of one or more outcomes
- Sample space: All possible outcomes for that experiment
- Probability of an event: A measure of the likelihood or chance that the event will occur
- Theoretical Probability:  $P_T(A) = \frac{O_F}{O_T} = \frac{\text{Favorable Outcomes}}{\text{Total Outcomes}}$
- Probability of the Complement of an Event:  $P(\bar{A}) = 1 - P(A)$
- Geometric Probabilities: Probabilities that can be found by calculating the ratio of two lengths, areas, or volumes
- Experimental Probability:  $P_E(A) = \frac{O_S}{O_T} = \frac{\text{Successful Outcomes}}{\text{Total Outcomes}}$

## II. Lesson 2: Independent and Dependent Events

- Independent Events: When the occurrence of one event does not affect the occurrence of the other event
- Dependent Events: When the occurrence of one event affects the occurrence of another event
- Probability of Independent Events

Words	The two events $A$ and $B$ are independent events if and only if the probability that both events occur is the product of the probabilities of the events
Symbols	$P(A \& B) = P(A) \times P(B)$

- Probability of Dependent Events

Words	If two events $A$ and $B$ are dependent events, then the probability that both events occur is the product of the probability of the first event and the conditional probability of the first event
Symbols	$P(A \& B) = P(A) \times P(B A)$

## III. Lesson 3: Two-Way Tables and Probability

- Two-Way Table: A frequency table that displays data collected from one source that belong to two different categories
- Joint Frequency: Each entry in the table
- Marginal Frequencies: The sums of rows and columns
- Relative and Conditional Relative Frequencies

Joint Relative Frequency	The ratio of a frequency that is not in the total row or the total column to the total number of values or observations
Marginal Relative Frequency	The sum of the joint relative frequencies in a row or column
Conditional Relative Frequency	The ratio of a joint relative frequency to the marginal relative frequency

- Conditional Probabilities:  $P(A|B) = \frac{P(A \& B)}{P(B)}$

## IV. Lesson 4: Probability of Disjoin and Overlapping Events

- Compound Event:
- Disjoint/Mutually Exclusive: No outcomes in common
- Overlapping: One or more outcomes in common
- Probability of Compound Events

Any two events	$P(A \cup B) = P(A) + P(B) - P(A \& B)$
Disjoint events	$P(A \cup B) = P(A) + P(B)$

## V. Lesson 5:

- N-factorial: The product of numbers from 1 to  $n$
- Permutations: An arrangement of objects in which order is important

Formulas	Examples
The number of permutations of $n$ objects is given by $nPn = n!$	The number of permutations of 4 objects is $4P4 = 4! = 4 \times 3 \times 2 \times 1$
The number of permutations of $n$ objects taken $r$ at a time, where $r \leq n$ , is given by $nPr = \frac{n!}{(n-r)!}$	The number of permutations of 4 objects taken 2 at a time $4P2 = \frac{4!}{(4-2)!} = \frac{4 \times 3}{1} = 12$

c. Combination: A selection of objects in which order is not important

Formula	Example
The number of combinations of $n$ objects taken $r$ at a time, where $r \leq n$ , is given by $nCr = \frac{n!}{(n-r)! \times r!}$	The number of combinations of 4 objects taken 2 at a time is $4C2 = \frac{4!}{(4-2)! \times 2!}$

d. The Binomial Theorem ( $r$  increases for each term)

$$(a + b)^n = (nC_r) a^{n-r} b^r$$

$r$  is an integer from 0 to  $n$

#### VI. Lesson 6: Binomial Distributions

a. Probability Distributions: A function that gives the probability of each possible value of a random variable

b. Binomial Experiments

- There are  $n$  independent trials

- Each trial has only two possible outcomes: Success or failure

- The probability of success is the same for each trial. This probability is denoted by  $p$

- The probability of failure is  $1 - p$

- Function of successes:  $P(k) = nC_k p^k (1 - p)^{n-k}$

# Chapter 11: Data Analysis and Statistics

Tuesday, March 2, 2021 9:29 PM

## I. Lesson 1: Using Normal Distributions

- a. Normal Distribution: A bell-shaped curve called a normal curve that is symmetric about the mean
- b. Areas Under a Normal Curve: A normal distribution with the mean  $\mu$  and standard deviation  $\sigma$  has these properties

Areas of $\mu - x\sigma$	Percentage
Total	100%
$x = 1$	68%
$x = 2$	95%
$x = 3$	99.7%

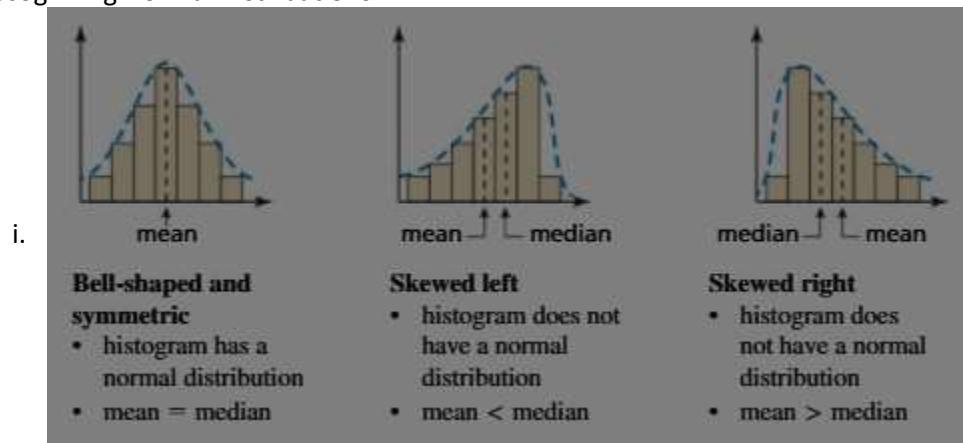
Areas of $\mu \pm x\sigma$	Percentage
$x = 1$	34%
$x = 2$	13.5%
$x = 3$	2.35%
$x = 4$	0.15%

- c. Standard Normal Distribution: The normal distribution with mean 0 and standard deviation 1. The formula below can be used to transform x-values from a normal distribution with mean  $\mu$  and standard deviation  $\sigma$  into z-values having a standard normal distribution

$$z = \frac{x - \mu}{\sigma}$$

$$x = \mu + z\sigma$$

- d. Recognizing Normal Distributions



## II. Lesson 2: Populations, Samples and Hypotheses

- a. Population: The collection of all data, such as responses, measurements or counts
- b. Sample: Subset of population
- c. Parameter and Statistics
  - i. Parameter: A numerical description of a population characteristic - All
  - ii. Statistic: Numerical description of a sample characteristic - Some
  - iii. Parameter and Statistic equation should ask who they are surveying

## III. Lesson 3: Collecting Data

- a. Random Sample: Preferred because it is most likely to be the representative of a population
- b. Types of Samples

Self-Selected Sample	Members of a population who volunteer
Systematic Sample	A rule is used to select members of a population
Stratified Sample	A population is divided into smaller group that share a similar characteristic, then a sample is randomly selected
Cluster Sample	A population is divided into groups, called clusters. All of the members in one or more clusters are selected
Convenience Sample	Only members who are easy to reach are selected

- c. Bias: Error that results in a misrepresentation of a population
- d. Unbiased Sample: Representative of the population that you want information about
- e. Biased Sample: A sample that overrepresents or under-represents part of the population
- f. Methods of Collecting Data

Experiment	A method that imposes a treatment on individuals in order to collect data on their response
Observational Study	Individuals are observed and variables are measured without controlling the individuals or their environment
Survey	An investigation of one or more characteristics of a population
Simulation	The use of a model to reproduce the conditions of a situation or process so that the simulated outcomes closely match the real-world outcomes

#### IV. Lesson 4: Experimental Design

- a. Controlled Experiment: Two groups are studies under ideal conditions with the exception of one variable
- b. Control group: The group is subject to no treatment
- c. Treatment group: The group that is subject to treatment
- d. Randomization: The process of randomly assigning subjects to different treatment groups
- e. Randomized Comparative Experiment: Subjects are randomly assigned to the control groups and treatment group - can make a valid cause and effect conclusions
- f. Placebo: A harmless, unmedicated treatment that resembles the actual treatment
- g. Experiments and Observational Studies
  - i. Experiments: Individuals are assigned at random to the treatment group or the control group
  - ii. Observational Study: When possible, random samples can be selected for the groups being studied - can identify correlation but not cause/effect
- h. Replication: The more times it is replicated, the more accurate

#### V. Lesson 5: Making Inferences from Sample Surveys

- a. Descriptive statistics: Involves the organization, summarization and display of data
- b. Inferential Statistics involves using a sample to draw conclusions about a population
- c. Margin of error:  $\pm \frac{1}{\sqrt{n}}$

#### VI. Lesson 6: Making Inferences from Experiments

- a. Resampling consists of rearranging all numbers in a complete set of data