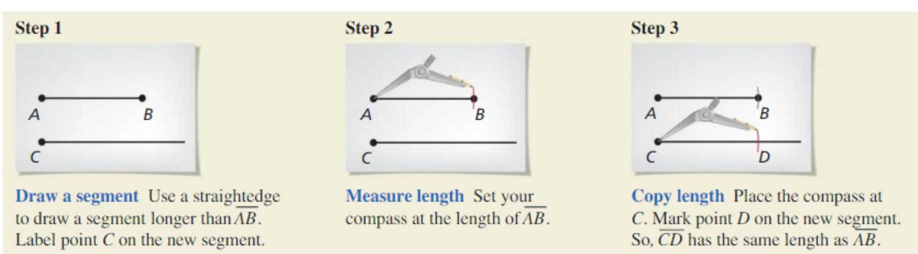
Finals 1 Study Guide

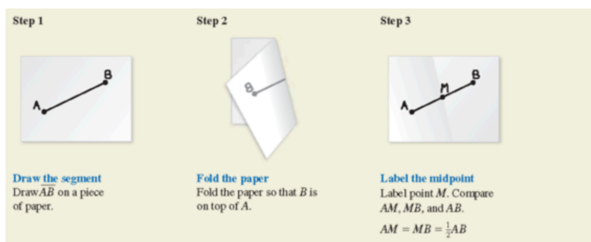
Chapters 1-5

I. Chapter 1: Basics of Geometry
  A. Lesson 1: Points, lines, planes
    1. Core Concepts
      a) Undefined Terms
        (1) Point: No dimension, represented by point
        (2) Line: One dimension, represented by a line with 2 arrowheads (extends without end)
        (3) Plane: 2 Dimension, presented by a shape which looks like a floor or wall, extends without end
      b) Defined Terms
        (1) Segments: Consists of endpoints A and B and all points on AB are between AB. AB can also be renamed BA
        (2) Rays: Consists of the endpoint A and all points on AB that lie on the same side of A as B
        (3) Opposite Rays: 2 Rays that are opposite of each other
    2. Vocab
      a) Collinear Points: Points that lie on same line
      b) Coplanar Points: Points that lie on the same plane
      c) Intersection: Set of points figures have in common
  B. Lesson 2: Measuring and Constructing segments
    1. Postulates
      a) Ruler Postulate: $|x_2-x_1|$
        (1) Distance between 2 points (vertical/horizontal)
      b) Segment Addition Postulate: AB + BC = AC
        (1) If AB + BC = AC, then B is between A & C
    2. Construction
      a) Copying a Segment

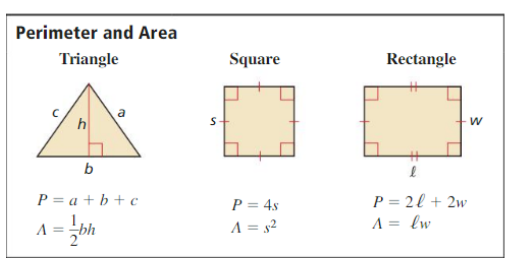


    3. Core Concepts
      a) Congruent Segments
        (1) Line segments that have the same length are called congruent segments
  C. Lesson 3: Using Midpoint and Distance formulas

1. Core Concepts
   a) Midpoints and Segment Bisectors
      (1) Midpoint: A point on a segment that divides the segment into two congruent segments
      (2) Segment Bisector: A point, ray, line, line segment, or plane that intersects the segment at its midpoint (it "bisects it")
   b) Using the Midpoint Formula
      (1) The Midpoint Formula: Coordinates of the midpoint of a segment are average of the two points
      $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$
   c) Using the Distance Formula
      (1) The Distance Formula: Finds distance between points
      $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
      (2)
2. Constructing
   a) Bisecting a Segment



D. Lesson 4: Perimeter and Area in the Coordinate Plane
1. Core Concepts
   a) Polygons: Closed plane figure with >3 line seg.
      (1) Classifying Polygons:
         (a) Convex: no line that contains side contains a point in the interior of polygon
         (b) Concave: not convex
2. Misc
   a) Coordinate Plane
      (1) Perimeter and Area



E. Lesson 5: Measuring and Constructing Angles
1. Core Concepts

a) Types of Angles
   (1) Acute: Measures from 0° and less than 90°
   (2) Right Angle: Measures 90°
   (3) Obtuse Angle: Measures greater than 90° and less than 180°
   (4) Straight Angle: Measures 180°

2. Vocab
   a) Angle: A set of points consisting of different rays
   b) Vertex: 2 rays common endpoint
   c) Interior Angle: The region that contains all points between the side of the angle
   d) Exterior Angle: Opposite of an Interior Angle
3. Postulates
   a) Protractor Postulate: The rays that form while measuring an angle can be matched with real numbers from 0 to 180

F. Lesson 6: Describing Pairs of Angles
   1. Core Concept
      a) Complementary and Supplementary Angles
         (1) Complementary Angles: 2 Angles with sum of 90°
         (2) Supplementary Angles: 2 Angles with sum of 180°
         (3) Adjacent Angles: 2 Angles that share a common vertex and side, but have no common interior points
      b) Linear Pairs and Vertical Angles
         (1) Linear Pair: Noncommon sides are opposite rays
         (2) Vertical angles: Sides form 2 pairs of opposite rays

II. Chapter 2:
   A. Lesson 1: Conditional Statements
      1. Core Concept
         a) Related Conditions
            (1) Words: Original (p-q)
            (2) Converse: The opposite(q-p)
            (3) Inverse: Negating both (~q-~p)
            (4) Contrapositive: Basically converse + inverse
         b) Biconditional Statements: A statement that includes the phrase "if and only if"
      2. Making Truth Tables
         a) Conditional Table

Conditional		
p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

b) Converse Table

Converse		
p	q	$q \to p$
T	T	T
T	F	T
F	T	F
F	F	T

c) Inverse Table

Inverse				
p	q	$\sim p$	$\sim q$	$\sim p \to \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

d) Contrapositive Table

Contrapositive				
p	q	$\sim q$	$\sim p$	$\sim q \to \sim p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

3. Vocab
    a) Perpendicular Lines: Two lines that intersect to form a right angle

B. Lesson 2: Inductive and Deductive Reasoning
1. Core Concept/Vocabulary
    a) Conjecture: Unproven statement based on observations
    b) Inductive Reasoning: A process that includes looking for patterns and making conjectures
    c) Counterexample: Specific case for which the conjecture is false
    d) Deductive Reasoning: A process that uses facts, definitions, accepted properties, and laws of logic to form a logical statement
        (1) Law of Detachment
            (a) If p is true, of a true conditional statement, the q is also true
        (2) Law of Syllogism
            (a) p -> q, q -> r, then p -> r

C. Lesson 3: Postulates and Diagrams

1. Point, Line, and Plane Postulates

Postulates

Point, Line, and Plane Postulates

Postulate / **Example**

2.1 Two Point Postulate

Through any two points, there exists exactly one line.

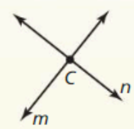
2.2 Line-Point Postulate

A line contains at least two points.

Through points *A* and *B*, there is exactly one line ℓ. Line ℓ contains at least two points.

2.3 Line Intersection Postulate

If two lines intersect, then their intersection is exactly one point.



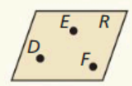
The intersection of line *m* and line *n* is point *C*.

2.4 Three Point Postulate

Through any three noncollinear points, there exists exactly one plane.

2.5 Plane-Point Postulate

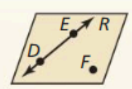
A plane contains at least three noncollinear points.



Through points *D*, *E*, and *F*, there is exactly one plane, plane *R*. Plane *R* contains at least three noncollinear points.

2.6 Plane-Line Postulate

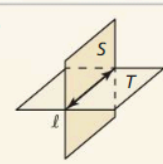
If two points lie in a plane, then the line containing them lies in the plane.



Points *D* and *E* lie in plane *R*, so $\overleftrightarrow{DE}$ lies in plane *R*.

2.7 Plane Intersection Postulate

If two planes intersect, then their intersection is a line.



The intersection of plane *S* and plane *T* is line ℓ.

D. Lesson 4: Algebraic Reasoning

1. Core Concepts

a) Distributive Property

(1) a(b + c) = ab + ac

(2) a(b - c) = ab - ac

b) Using Other Properties of Equality

Reflexive, Symmetric, and Transitive Properties of Equality

	Real Numbers	Segment Lengths	Angle Measures
Reflexive Property	$a = a$	$AB = AB$	$m\angle A = m\angle A$
Symmetric Property	If $a = b$, then $b = a$.	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

E. Lesson 5: Proving Statements about Segments and Angles

1. Vocabulary
    a) Proof: Logical argument that uses deductive reasoning to show a statement is true
    b) Two-column proof: A type of proof that has numbers statements and corresponding reasons that show an argument in a logical order
    c) Theorem: statement that can be proven

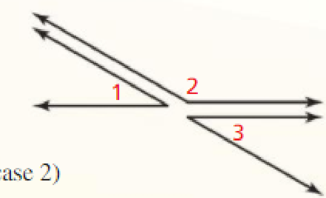
F. Lesson 6: Proving Geometric Relationships

1. Theorems
    a) Right Angles Theorem: All right angles are congruent
    b) Congruent Supplements Theorems

Theorem 2.4 Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.



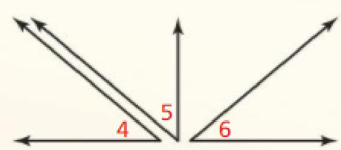
Proof Example 2, p. 107 (case 1); Ex. 20, p. 113 (case 2)

    c) Congruent Complements Theorem

Theorem 2.5 Congruent Complements Theorem

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then $\angle 4 \cong \angle 6$.

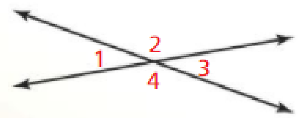


Proof Ex. 19, p. 112 (case 1); Ex. 22, p. 113 (case 2)

d) Vertical Angles Congruence Theorem

Theorem 2.6 Vertical Angles Congruence Theorem

Vertical angles are congruent.



Proof Example 3, p. 108

$\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$

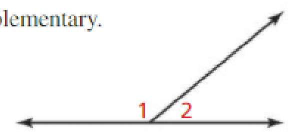
2. Postulates

a) Linear Pair Postulate

Postulate 2.8 Linear Pair Postulate

If two angles form a linear pair, then they are supplementary.

$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180°$.



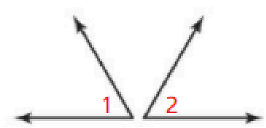
3. Concept Summary

Types of Proofs

Symmetric Property of Angle Congruence (Theorem 2.2)

Given $\angle 1 \cong \angle 2$

Prove $\angle 2 \cong \angle 1$



Two-Column Proof

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $m\angle 1 = m\angle 2$	**2.** Definition of congruent angles
3. $m\angle 2 = m\angle 1$	**3.** Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	**4.** Definition of congruent angles

Flowchart Proof

$\angle 1 \cong \angle 2$ → $m\angle 1 = m\angle 2$ → $m\angle 2 = m\angle 1$ → $\angle 2 \cong \angle 1$

Given; Definition of congruent angles; Symmetric Property of Equality; Definition of congruent angles

Paragraph Proof

$\angle 1$ is congruent to $\angle 2$. By the definition of congruent angles, the measure of $\angle 1$ is equal to the measure of $\angle 2$. The measure of $\angle 2$ is equal to the measure of $\angle 1$ by the Symmetric Property of Equality. Then by the definition of congruent angles, $\angle 2$ is congruent to $\angle 1$.

III. Chapter 3: Parallel and Perpendicular Lines

A. Lesson 1: Pairs of Lines and Angles

1. Core Concept

a) Parallel Lines, Skew Lines, Parallel Planes

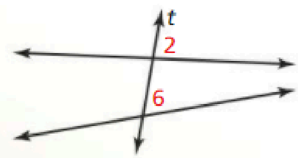
(1) Parallel Lines: Two lines that don't intersect and are coplanar

(2) Skew Lines: Two lines that don't intersect and aren't coplaner lines

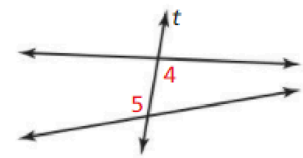
(3) Parallel Planes: 2 Planes that don't intersect

b) Identifying Pairs of Angles

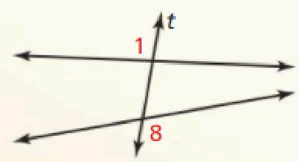
Angles Formed by Transversals



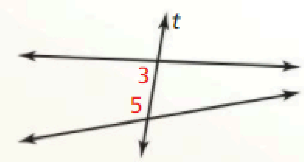
Two angles are **corresponding angles** when they have corresponding positions. For example, ∠2 and ∠6 are above the lines and to the right of the transversal *t*.



Two angles are **alternate interior angles** when they lie between the two lines and on opposite sides of the transversal *t*.



Two angles are **alternate exterior angles** when they lie outside the two lines and on opposite sides of the transversal *t*.



Two angles are **consecutive interior angles** when they lie between the two lines and on the same side of the transversal *t*.

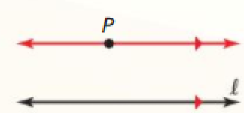
2. Postulates

a) Parallel Postulate

Postulate 3.1 Parallel Postulate

If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.

There is exactly one line through *P* parallel to ℓ.

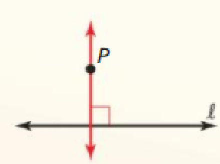


b) Perpendicular Postulate

Postulate 3.2 Perpendicular Postulate

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

There is exactly one line through *P* perpendicular to ℓ.



B. Lesson 2: Parallel Lines and Transversals

1. Properties of Parallel Lines

Theorems

Theorem 3.1 Corresponding Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Examples In the diagram at the left, $\angle 2 \cong \angle 6$ and $\angle 3 \cong \angle 7$.

Proof Ex. 36, p. 180

Theorem 3.2 Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Examples In the diagram at the left, $\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$.

Proof Example 4, p. 134

Theorem 3.3 Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

Examples In the diagram at the left, $\angle 1 \cong \angle 8$ and $\angle 2 \cong \angle 7$.

Proof Ex. 15, p. 136

Theorem 3.4 Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

Examples In the diagram at the left, $\angle 3$ and $\angle 5$ are supplementary, and $\angle 4$ and $\angle 6$ are supplementary.

Proof Ex. 16, p. 136

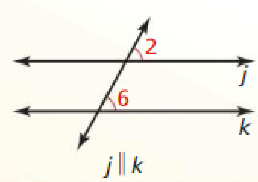
C. Lesson 3: Proofs with Parallel Lines

1. Theorem

Theorem

Theorem 3.5 Corresponding Angles Converse

If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.

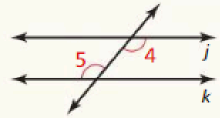


Proof Ex. 36, p. 180

Theorems

Theorem 3.6 Alternate Interior Angles Converse

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

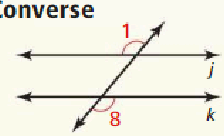


$j \parallel k$

Proof Example 2, p. 140

Theorem 3.7 Alternate Exterior Angles Converse

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

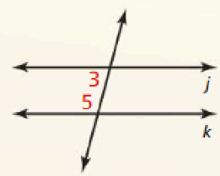


$j \parallel k$

Proof Ex. 11, p. 142

Theorem 3.8 Consecutive Interior Angles Converse

If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.



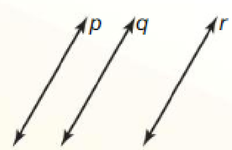
If $\angle 3$ and $\angle 5$ are supplementary, then $j \parallel k$.

Proof Ex. 12, p. 142

Theorem

Theorem 3.9 Transitive Property of Parallel Lines

If two lines are parallel to the same line, then they are parallel to each other.



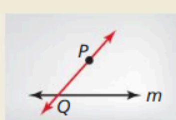
If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

Proof Ex. 39, p. 144; Ex. 48, p. 162

2.

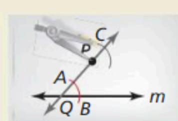
1. Constructing Parallel Lines

Step 1



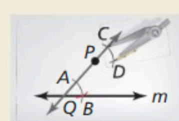
Draw a point and line Start by drawing point *P* and line *m*. Choose a point *Q* anywhere on line *m* and draw $\overleftrightarrow{QP}$.

Step 2



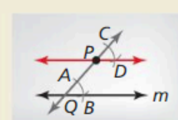
Draw arcs Draw an arc with center *Q* that crosses $\overleftrightarrow{QP}$ and line *m*. Label points *A* and *B*. Using the same compass setting, draw an arc with center *P*. Label point *C*.

Step 3



Copy angle Draw an arc with radius *AB* and center *A*. Using the same compass setting, draw an arc with center *C*. Label the intersection *D*.

Step 4

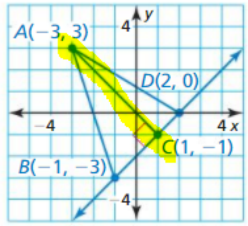


Draw parallel lines Draw $\overleftrightarrow{PD}$. This line is parallel to line *m*.

D. Lesson 4: Proofs with Perpendicular Lines

1. Distance from Point to a Line

a) The length of the perpendicular segment from a point to the line

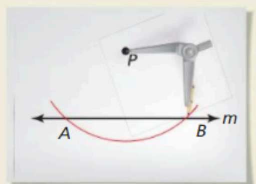


(1) The length of AC is the distance from BD

2. Construction

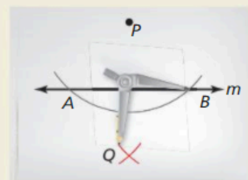
a) Constructing a Perpendicular Line

Step 1



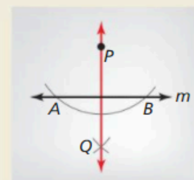
Draw arc with center *P* Place the compass at point *P* and draw an arc that intersects the line twice. Label the intersections *A* and *B*.

Step 2



Draw intersecting arcs Draw an arc with center *A*. Using the same radius, draw an arc with center *B*. Label the intersection of the arcs *Q*.

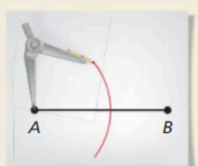
Step 3



Draw perpendicular line Draw $\overleftrightarrow{PQ}$. This line is perpendicular to line *m*.

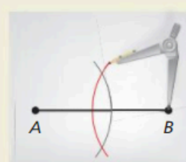
b) Constructing a Perpendicular Bisector

Step 1



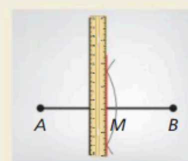
Draw an arc Place the compass at *A*. Use a compass setting that is greater than half the length of $\overline{AB}$. Draw an arc.

Step 2



Draw a second arc Keep the same compass setting. Place the compass at *B*. Draw an arc. It should intersect the other arc at two points.

Step 3



Bisect segment Draw a line through the two points of intersection. This line is the perpendicular bisector of $\overline{AB}$. It passes through *M*, the midpoint of $\overline{AB}$. So, $AM = MB$.

3. Theorem

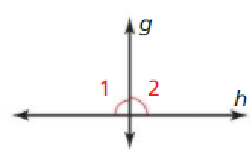
a) Linear Pair Perpendicular Theorem

Theorem 3.10 Linear Pair Perpendicular Theorem

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

If $\angle 1 \cong \angle 2$, then $g \perp h$.

Proof Ex. 13, p. 153



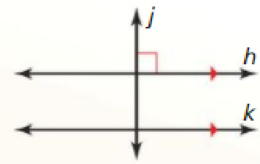
b) Perpendicular Transversal Theorem

Theorem 3.11 Perpendicular Transversal Theorem

In a plane, if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.

If $h \parallel k$ and $j \perp h$, then $j \perp k$.

Proof Example 2, p. 150; Question 2, p. 150



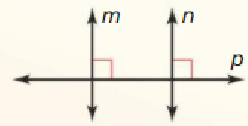
c) Lines Perpendicular to a Transversal Theorem

Theorem 3.12 Lines Perpendicular to a Transversal Theorem

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

If $m \perp p$ and $n \perp p$, then $m \parallel n$.

Proof Ex. 14, p. 153; Ex. 47, p. 162



E. Lesson 5: Equations of Parallel and Perpendicular Lines

1. Partitioning a Directed Line Segment
   a) Get ratio into fraction form
      (1) Partition Ratio (Add ratio)
      (2) Find fraction form of the distance from the 2 points
   b) Find rise and run, do not form it into a fraction
   c) Multiply rise and run by fraction
   d) Add the finished rise and run to the first point (rise goes to y, run goes to x)
2. Theorems
   a) Identifying Parallel and Perpendicular Lines

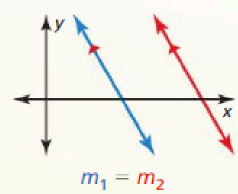
Theorems

Theorem 3.13 Slopes of Parallel Lines

In a coordinate plane, two distinct nonvertical lines are parallel if and only if they have the same slope.

Any two vertical lines are parallel.

Proof p. 439; Ex. 41, p. 444

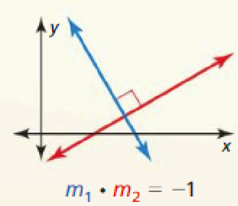


Theorem 3.14 Slopes of Perpendicular Lines

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Horizontal lines are perpendicular to vertical lines.

Proof p. 440; Ex. 42, p. 444



IV. Chapter 4: Transformations

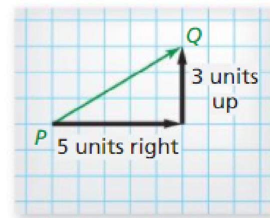
A. Lesson 1: Translations

1. Core Concepts

a) Vectors

Vectors

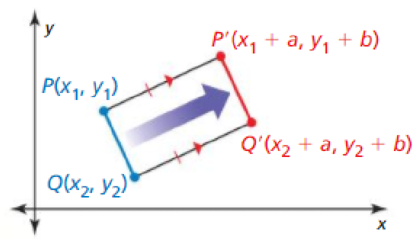
The diagram shows a vector. The **initial point**, or starting point, of the vector is P, and the **terminal point**, or ending point, is Q. The vector is named $\overrightarrow{PQ}$, which is read as "vector PQ." The **horizontal component** of $\overrightarrow{PQ}$ is 5, and the **vertical component** is 3. The **component form** of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{PQ}$ is $\langle 5, 3 \rangle$.



b) Translation

Translations

A **translation** moves every point of a figure the same distance in the same direction. More specifically, a translation *maps*, or moves, the points P and Q of a plane figure along a vector $\langle a, b \rangle$ to the points P' and Q', so that one of the following statements is true.



- $PP' = QQ'$ and $\overline{PP'} \parallel \overline{QQ'}$, or
- $PP' = QQ'$ and $\overline{PP'}$ and $\overline{QQ'}$ are collinear.

2. Postulates

a) Translation Postulate

Postulate 4.1 Translation Postulate

A translation is a rigid motion.

b) Composition Theorem

Theorem 4.1 Composition Theorem

The composition of two (or more) rigid motions is a rigid motion.

Proof Ex. 35, p. 180

B. Lesson 2: Reflections

1. Core Concepts

a) Reflection: transformation using line like a mirror to reflect a figure

b) Coordinate Rules for Reflection

(1) If (a, b) is reflected in the x-axis, then its image is the point (a, −b).

(2) If (a, b) is reflected in the y-axis, then its image is the point (−a, b).

(3) If (a, b) is reflected in the line y = x, then its image is the point (b, a).

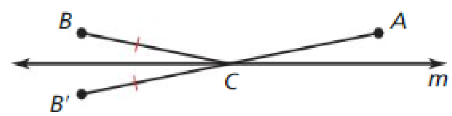
(4) If (a, b) is reflected in the line y = −x, then its image is the point (−b, −a).

2. Postulates
    a) Reflection Postulate

Postulate

Postulate 4.2 Reflection Postulate
A reflection is a rigid motion.

3. Identifying Lines of Symmetry
    a) Line symmetry: figure can be mapped onto itself by reflection in line
    b) Line of symmetry: the line of reflection
4. Finding Minimum distance
    a) Reflect B in line m to obtain B'. Then draw $\overline{AB'}$. Label the intersection of $\overline{AB'}$ and m as C. Because AB' is the shortest distance between A and B' and $BC = B'C$, park at point C to minimize the combined distance, $AC + BC$, you both have to walk.

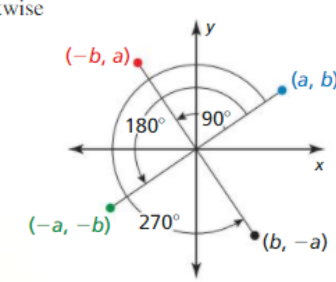


C. Lesson 3: Rotations
1. Core Concepts
    a) Performing Rotation
        (1) Rotation: A transformation in which a figure is turned about a fixed point known as the center of rotation
        (2) Angle of rotation: Rays drawn from the from the center of rotation to a point and its image
    b) Rotating around Origin

Coordinate Rules for Rotations about the Origin

When a point (a, b) is rotated counterclockwise about the origin, the following are true.

- For a rotation of 90°, $(a, b) \rightarrow (-b, a)$.
- For a rotation of 180°, $(a, b) \rightarrow (-a, -b)$.
- For a rotation of 270°, $(a, b) \rightarrow (b, -a)$.



2. Postulates
    a) Rotation Postulate

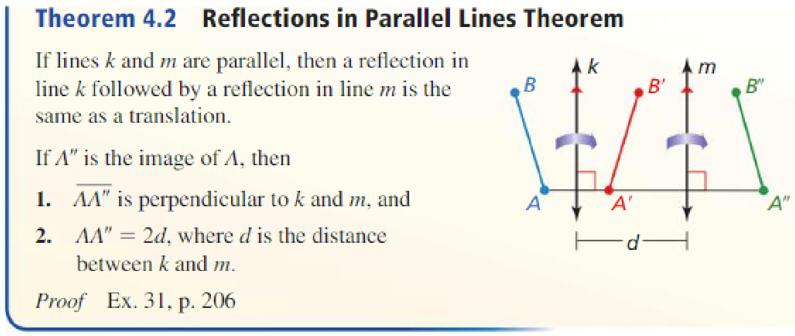
Postulate 4.3 Rotation Postulate
A rotation is a rigid motion.

D. Lesson 4: Congruence and Transformations
1. Vocabulary
    a) Congruent figures: there is a rigid motion or composition of rigid motions that maps one of the figures onto the other

b) Congruence transformation: another name for rigid motion or combo of rigid motion

2. Theorem

a) Reflections in Parallel Lines Theorem



Theorem 4.2 Reflections in Parallel Lines Theorem

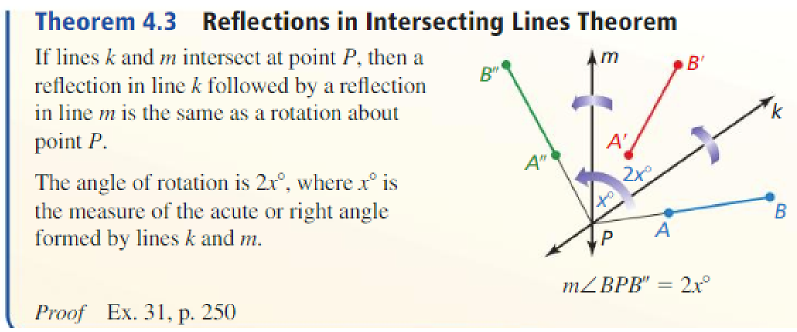
If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is the same as a translation.

If A'' is the image of A, then

1. $\overline{AA''}$ is perpendicular to k and m, and
2. $AA'' = 2d$, where d is the distance between k and m.

Proof Ex. 31, p. 206

b) Reflections in Intersecting Lines Theorem



Theorem 4.3 Reflections in Intersecting Lines Theorem

If lines k and m intersect at point P, then a reflection in line k followed by a reflection in line m is the same as a rotation about point P.

The angle of rotation is $2x°$, where $x°$ is the measure of the acute or right angle formed by lines k and m.

Proof Ex. 31, p. 250

E. Lesson 5: Dilations

1. Core Concepts

a) Dilation: transformation in which a figure is enlarged or reduced

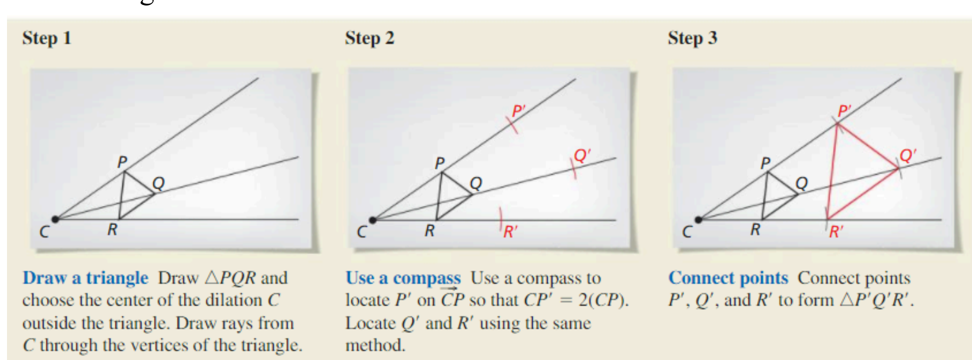
b) Center of Dilation: the center of dilation

c) Scale factor: ratio of the lengths of corr. Sides of image and preimage

d) Coordinate Rule for Dilations: If dilation is centered at the origin then (kx,ky) is the image

2. Construction

a) Constructing a dilation



Draw a triangle Draw $\triangle PQR$ and choose the center of the dilation C outside the triangle. Draw rays from C through the vertices of the triangle.

Use a compass Use a compass to locate P' on $\overrightarrow{CP}$ so that $CP' = 2(CP)$. Locate Q' and R' using the same method.

Connect points Connect points P', Q', and R' to form $\triangle P'Q'R'$.

F. Lesson 6: Similarity and Transformations

1. Vocab

a) Similarity Transformation: A dilation or a composition of rigid motions and dilations

b) Similar figures: Geometric figures that have the same shape but not necessarily the same size

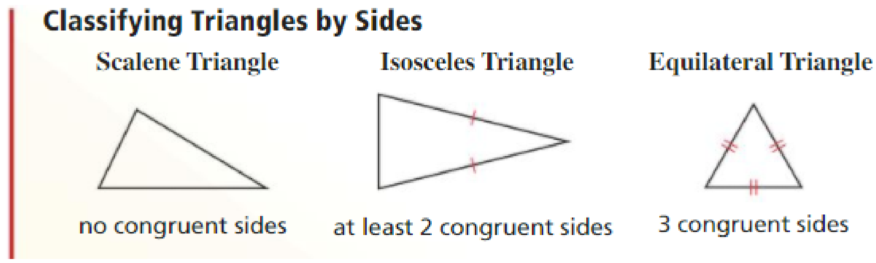
2.

V. Chapter 5

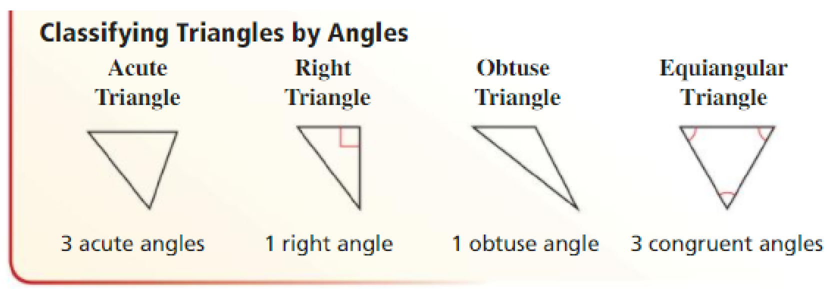
A. Lesson 1: Angles of Triangles

1. Core Concept

a) Classifying Triangles by Sides

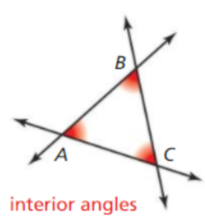


b) Classifying Triangles by angles

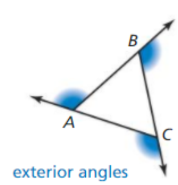


2. Vocabulary

a) Interior Angles: Original Angles



b) Exterior Angles: Angles that form linear pairs with interior angles

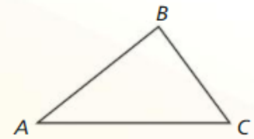


3. Theorems

a) Triangle Sum Theorem

Theorem 5.1 Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.



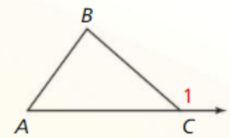
$m\angle A + m\angle B + m\angle C = 180°$

Proof p. 234; Ex. 53, p. 238

b) Exterior Angles Theorem

Theorem 5.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.



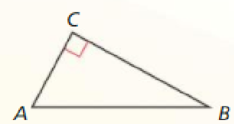
$m\angle 1 = m\angle A + m\angle B$

Proof Ex. 42, p. 237

c) Corollary to the Triangle Sum Theorem

Corollary 5.1 Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.



$m\angle A + m\angle B = 90°$

Proof Ex. 41, p. 237

B. Lesson 2: Congruent Polygons

1. Vocabulary
   a) Corresponding part (angles and lengths): a part that is mapped on by rigid motions
   b) If all corresponding parts are congruent, then the triangles are congruent
2. Theorems

Theorem 5.3 Properties of Triangle Congruence

Triangle congruence is reflexive, symmetric, and transitive.

Reflexive For any triangle $\triangle ABC$, $\triangle ABC \cong \triangle ABC$.

Symmetric If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.

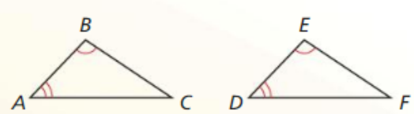
Transitive If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

Proof BigIdeasMath.com

a)

Theorem 5.4 Third Angles Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.



If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

Proof Ex. 19, p. 244

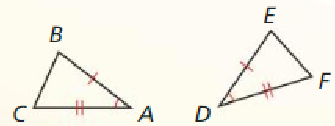
C. Lesson 3: Proving TriangleCongruenceby SAS

1. SAS congruence theorem

Theorem 5.5 Side-Angle-Side (SAS) Congruence Theorem

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

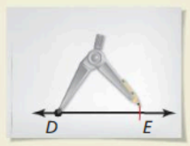
If $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.



Proof p. 246

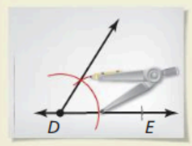
2. Copying a triangle using SAS

Step 1



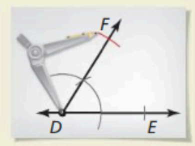
Construct a side
Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$.

Step 2



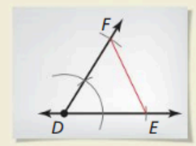
Construct an angle
Construct $\angle D$ with vertex D and side $\overrightarrow{DE}$ so that it is congruent to $\angle A$.

Step 3



Construct a side
Construct $\overline{DF}$ so that it is congruent to $\overline{AC}$.

Step 4



Draw a triangle
Draw $\triangle DEF$. By the SAS Congruence Theorem, $\triangle ABC \cong \triangle DEF$.

D. Lesson 4: Equilateral and Isosceles Triangles

1. Theorems

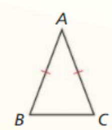
a) Base Angles Theorem

Theorem 5.6 Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.

Proof p. 252



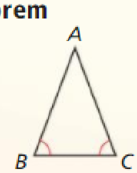
b) Converse of Base Angles Theorem

Theorem 5.7 Converse of the Base Angles Theorem

If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.

Proof Ex. 27, p. 275



2. Collieries

a) Corollary to the Base Angles Theorem

Corollary 5.2 Corollary to the Base Angles Theorem

If a triangle is equilateral, then it is equiangular.

Proof Ex. 37, p. 258; Ex. 10, p. 353

b) Corollary to the Converse of Base Angles Theorem

Corollary 5.3 Corollary to the Converse of the Base Angles Theorem

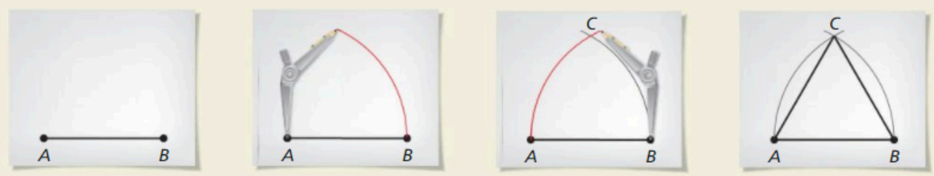
If a triangle is equiangular, then it is equilateral.

Proof Ex. 39, p. 258

3. Construction

a) Constructing an Equilateral Triangle

Step 1 **Step 2** **Step 3** **Step 4**



Copy a segment Copy $\overline{AB}$.

Draw an arc Draw an arc with center A and radius AB.

Draw an arc Draw an arc with center B and radius AB. Label the intersection of the arcs from Steps 2 and 3 as C.

Draw a triangle Draw $\triangle ABC$. Because $\overline{AB}$ and $\overline{AC}$ are radii of the same circle, $\overline{AB} \cong \overline{AC}$. Because $\overline{AB}$ and $\overline{BC}$ are radii of the same circle, $\overline{AB} \cong \overline{BC}$. By the Transitive Property of Congruence (Theorem 2.1), $\overline{AC} \cong \overline{BC}$. So, $\triangle ABC$ is equilateral.

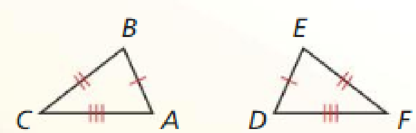
E. Lesson 5: Proving Triangle Congruence By SSS

1. Theorems

Theorem 5.8 Side-Side-Side (SSS) Congruence Theorem

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

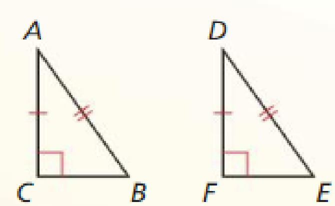
If $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.



Theorem 5.9 Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

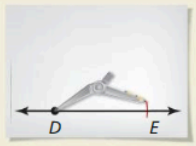
If $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $m\angle C = m\angle F = 90°$, then $\triangle ABC \cong \triangle DEF$.



Proof Ex. 38, p. 470; *BigIdeasMath.com*

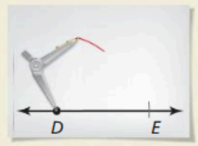
2. Copying a Triangle Using SSS

Step 1



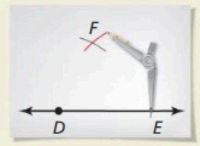
Construct a side
Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$.

Step 2



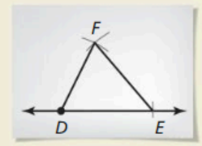
Draw an arc
Open your compass to the length AC. Use this length to draw an arc with center D.

Step 3



Draw an arc
Draw an arc with radius BC and center E that intersects the arc from Step 2. Label the intersection point F.

Step 4



Draw a triangle
Draw $\triangle DEF$. By the SSS Congruence Theorem, $\triangle ABC \cong \triangle DEF$.

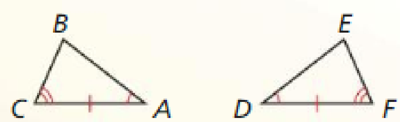
F. Lesson 6: Proving Triangle Congruence by ASA and AAS

1. Theorems

Theorem 5.10 Angle-Side-Angle (ASA) Congruence Theorem

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

If $\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$, and $\angle C \cong \angle F$, then $\triangle ABC \cong \triangle DEF$.

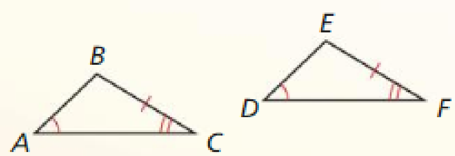


Proof p. 270

Theorem 5.11 Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

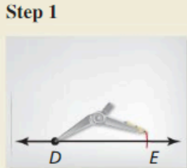
If $\angle A \cong \angle D$, $\angle C \cong \angle F$, and $\overline{BC} \cong \overline{EF}$, then $\triangle ABC \cong \triangle DEF$.



Proof p. 271

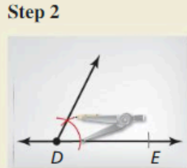
2. Copying a Triangle Using ASA

Step 1



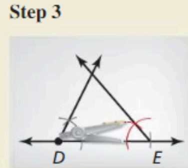
Construct a side
Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$.

Step 2



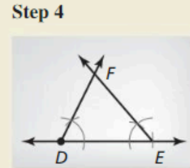
Construct an angle
Construct $\angle D$ with vertex D and side $\overrightarrow{DE}$ so that it is congruent to $\angle A$.

Step 3



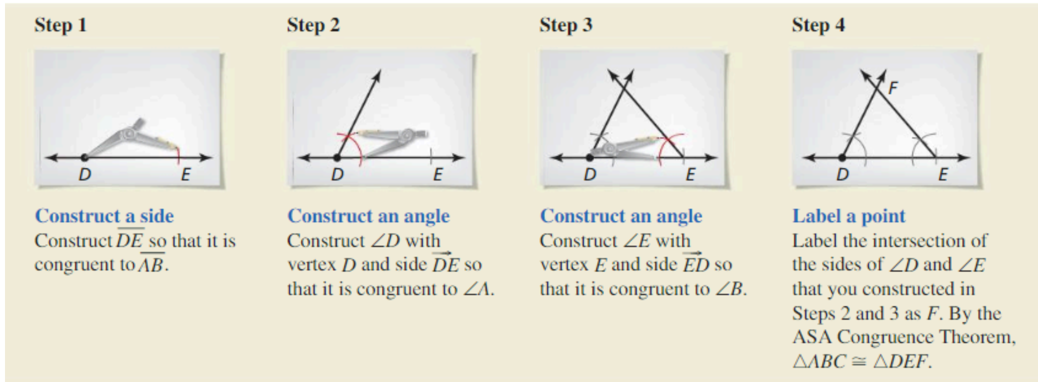
Construct an angle
Construct $\angle E$ with vertex E and side $\overrightarrow{ED}$ so that it is congruent to $\angle B$.

Step 4



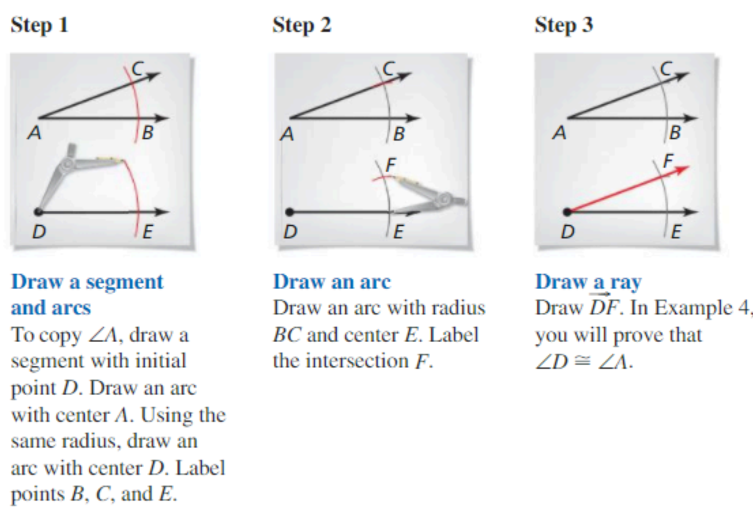
Label a point
Label the intersection of the sides of $\angle D$ and $\angle E$ that you constructed in Steps 2 and 3 as F. By the ASA Congruence Theorem, $\triangle ABC \cong \triangle DEF$.

3. Summary of all triangle congruence theorems



G. Lesson 7 Using Congruent Triangles

1. Proving congruent triangles
    a) Use corresponding parts to prove congruent triangles
2. Proving Constructions



H. Lesson 8: Coordinate Proofs

1. Placing Figures in a Coordinate Plane
    a) Usually would place figure at origin
2. Coordinate Proof: placing geometric figures in a coordinate plane to prove